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Analysis of a focus log electrode in a non homogeneous medium

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ANALYSIS OF A FOCUS LOG ELECTRODE IN A NON HOMOGENEOUS MEDIUM

by

JOHN JULIAN ZENOR, 1943

A DISSERTATION

Presented to the Faculty of the Graduate School of the

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DOCTOR OF PHILOSOPHY

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ABSTRACT

The mathematical model presented for the focused logging system is applicable to any resistance logging tool of the electrode type. Horizontal or cylindrical resistance interfaces may be included in any combinations.

The tool and its environment are compartmentalized, and the integral form of the condition for continuity of current is applied to each compartment. Acceleration methods are applied to the iterative solution of the resulting large-scale system of linear equations.

Two specific models are presented, one for the study of the effects of invasion and the other for the study of the effects of bed thickness on the apparent resistivity readings of the tool.

INTRODUCTION

Quantitative interpretation of electric well logs for the determination of reservoir properties such as porosity and water saturation was long delayed after the introduction of resistance well logging because of the difficulty of obtaining reliable estimates for true resistivity, R_t.

The first step in this direction was taken in May 1947 when the first resistivity departure curves were introduced by Schlumberger. In 1948 the Microlog was introduced by Schlumberger as the first attempt to facilitate quantitative interpretation by determining the resistivity of the invaded zone.

The combination of tools having different characteristics for the determination of R_t, the resistance and depth of the invaded zone, and other pertinent properties for the ultimate determination of reservoir parameters such as saturation has received much recent interest.

The use of these combinations, however, depends on the availability of suitable departure curves giving detailed information on the response of the tool to its environment.

One combination recently investigated is that of the induction well log with the focus or guard log. Unfortunately, although analysis for the induction log has reached an advanced stage, analysis for the focused log has not been adequate.

Most of the analysis of resistance well logging tools has been limited to point electrode systems where analytical solutions are known.

The potential distribution of a point electrode in a layered earth has been available since 1930 (Stefanesco, 1930); however, the computations involved are quite lengthy. Many methods of computational improvement have been suggested (e.g., Flathe, 1955, Unz, 1962, Onodera, 1963, and Nabighian, 1966). The method suggested by Nabighian uses finite forward differences of higher orders and repeated summation by parts to accelerate the convergence of the potential and resistivity series.

Owen and Greer (1951) advanced an analytical method for the guard electrode logging system, a system where the electrodes have large surface areas and may not be adequately described as point sources. This method approximates the tool, the borehole, and the invaded zone as prolate spheroids with the minor diameter set to the dimension of the corresponding surface.

It was pointed out in a later publication (De Witte, _et al._, 1957) that the prolate spheroid approximation, in which the equipotential surfaces are prolate spheroids, breaks down for the case where the resistivity of the drill hole fluid is much smaller than the formation resistivity. This is due to the refraction of the current flow at the cylindrical resistance interfaces whereas the prolate spheriod approximation yields hyperbolic surfaces of flow. A method is then presented approximating the electrodes by a series of spheres having the same diameter as the electrode.

The basic problem with methods based on analytical solutions for the point or spherical electrode systems is that the analysis is limited to environments where the analytical solutions are known. For example, no adequate solution exists for the layered earth which is penetrated by a borehole.

The mathematical model of the focused logging tool presented here is applicable to any resistance well logging tool of the electrode type, with any configuration of cylindrical and horizontal resistance boundaries symmetric about the axis of the tool.

The solution to the model is obtained by compartmentalizing the tool and its surrounding media and then applying the integral form of the condition for continuity of current to each compartment. Acceleration methods are applied to the iterative solution of the resulting large-scale system of otherwise slowly convergent linear equations in order to yield economic solutions.

Two specific models are presented as examples: one for the study of the effects of an invaded zone and the other for the study of the effects of bed thickness on apparent resistivity.

CHAPTER I

MODELS FOR THIN BED AND INVASION STUDIES

WELL LOGGING, GENERAL

In electric well logging, a sonde containing the electrodes for measuring various electrical properties of the earth is lowered into the well on an armored cable. This cable contains the conductors to connect the devices in the sonde to recording equipment at the surface. At the surface continuous recording is made of the outputs of these measuring devices plotted against depth as the sonde is withdrawn from the hole.

Ideally, these measuring devices would give outputs corresponding to the true electrical properties of the formation surrounding the borehole. Their outputs, however, are influenced by the presence of the borehole, the invasion of the mud filtrate into the formation, and even by the presence of the sonde itself. Thus, only apparent values of the electrical properties are recorded at the surface, and the true values must be obtained by applying corrections to these apparent values. This can only be accomplished after a detailed knowledge of the tool's environment and of its behavior in this environment is obtained.

The mud in the hole is made heavy enough to prevent formation fluids from penetrating into the borehole. However, if the borehole passes through a region of high permeability, the mud filtrate will still penetrate into the formation, producing an invaded zone. The invasion of filtrate from the borehole into the formation quickly results in the formation of a mud cake on the inside of the borehole which limits the loss of fluid. The actual depth of penetration will depend upon factors such as time, filtration properties of the mud, porosity of the formation, and pressure differential between the mud formation fluids.

The diameter of penetration may be assumed to vary from twice that of the borehole for high-porosity formations to ten times that of the borehole for low-porosity formations.

For low-permeability beds, such as shale, little or no invasion takes place, and no mud cake forms.

RESISTANCE WELL LOGGING

The subsurface electrode system for the focus log is shown in Figure 1A. The device is used to measure resistivity in a narrowly focused region normal to the center of the electrode. There are many other electrode systems of a nonfocused type, such as those used for the normal and lateral logs shown diagrammatically in Figures 1B and 1C.

The measurements of formation resistivity are affected by the mud in the borehole, the invasion of the mud filtrate into the formation, and the thickness of the bed for which the resistivity is being determined. Thus, the readings taken from a resistance well log are not the true formation resistivities but instead are apparent resistivities.

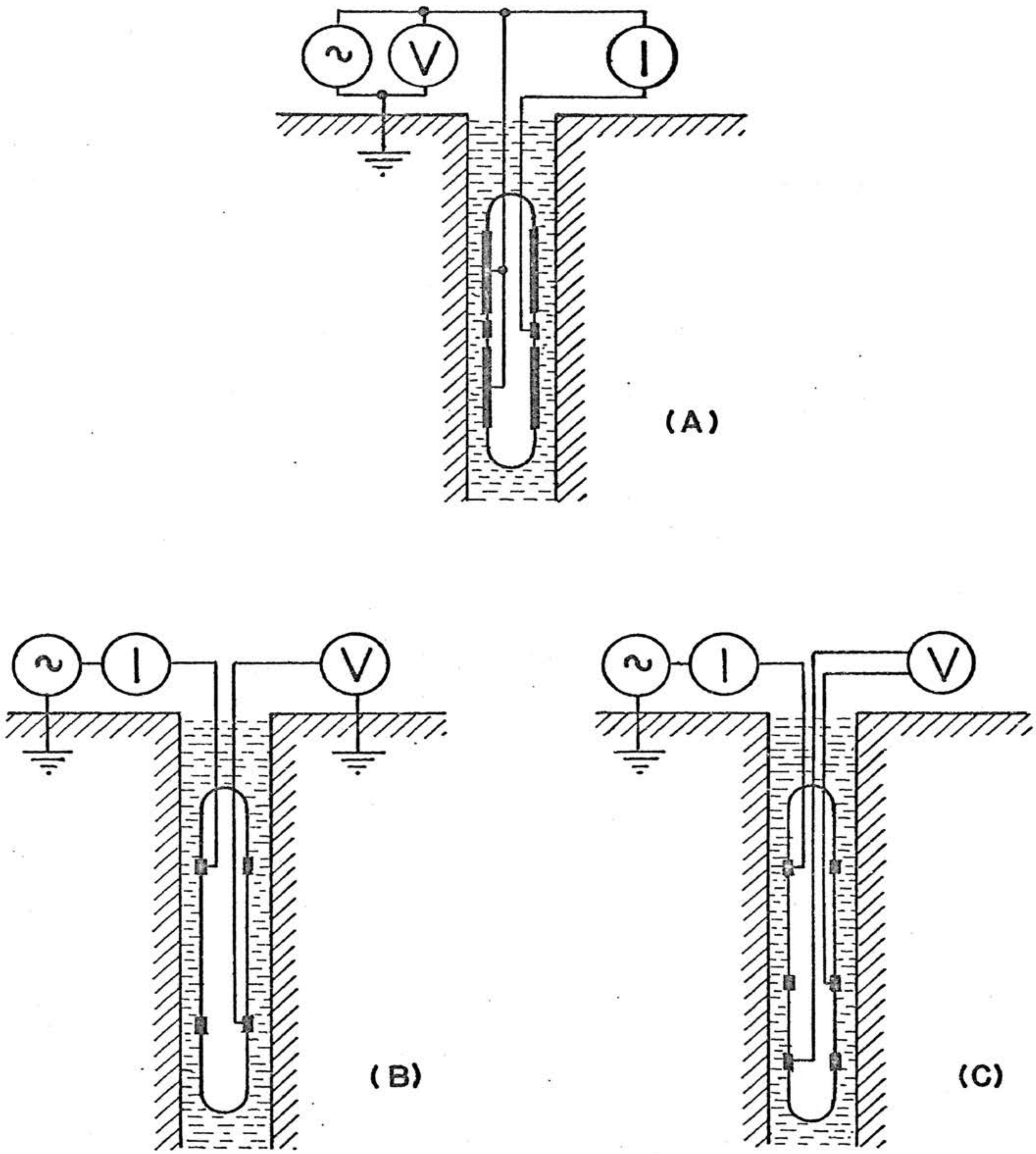


FIG. 1. ELECTRODE SYSTEMS. (A) FOCUS LOG (B) NORMAL LOG (C) LATERAL LOG.

If all the physical parameters involved were known, the appropriate corrections could be made to the apparent resistivity values to obtain the true resistivities. Curves showing the functional relationship between these physical parameters and the effects of these parameters on the reading of the logging device are known as departure curves.

Since the parameters, such as invasion depth, are seldom precisely known, they must be inferred from a set of logging measurements made by using devices which have characteristics different from the parameters being determined. To accomplish this one must know the characteristics of the tools being employed.

A possible combination of tools is the focus log with the induction well log. The characteristics of the induction well logging tool are known in some detail, but no adequate mathematical treatment of the focus logging tool has been developed heretofore.

A mathematical model of the focus logging tool and its surroundings is introduced with a numerical method of solution for the model. Although the focus logging tool is modeled, the method is applicable to any resistance logging tool of the electrode type.

FOCUS LOGGING TOOL

A dimensioned drawing of the tool used in the model is shown in Figure 2. The tool is composed basically of two guard electrodes separated by insulators from a central measure electrode. The two guard electrodes and the measure electrode are held at a constant

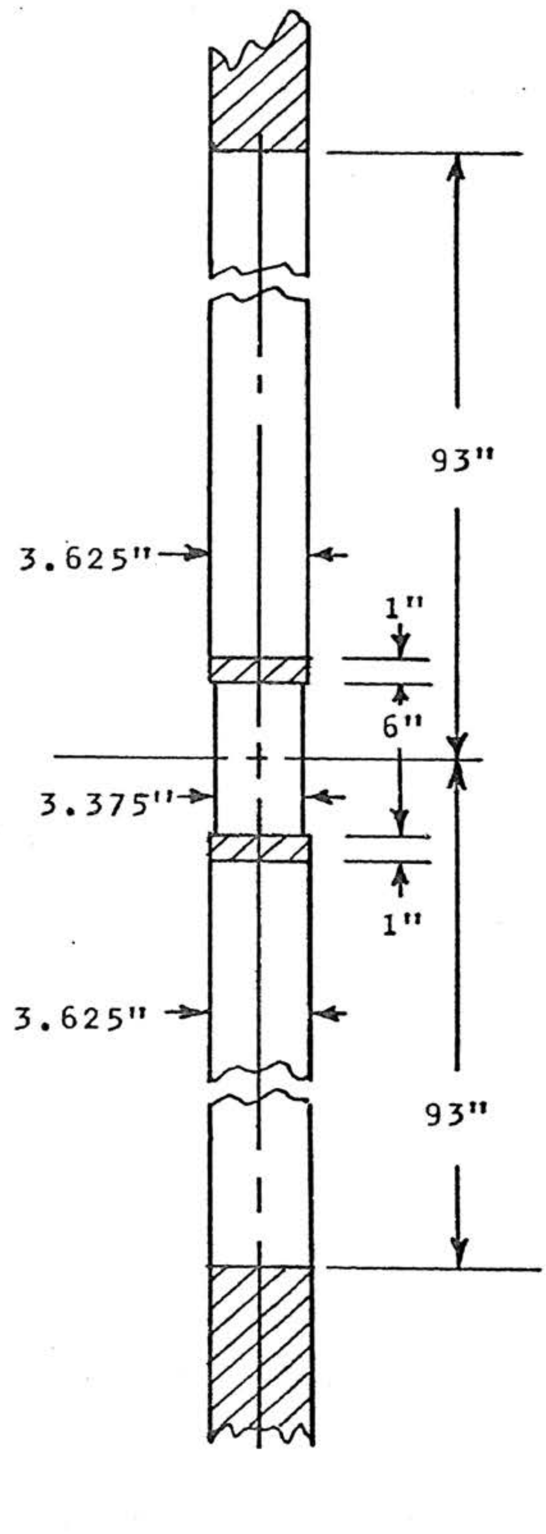


Fig. 2. Focus log electrode used in the examples.

potential V. The apparent resistivity R_a of the surrounding medium is then given by

$$R_a = k\ V/I \tag{1}$$

where k is the tool constant and I is the current flowing from the measure electrode.

The function of the guard electrodes is to keep the current flow nearly radial at the measure electrode, improving the tool's performance in thin beds.

BASIC MODELS

Models were constructed to study the effects of invasion and the effects of bed thickness on the tool response. The bed structure was assumed to be perpendicular to the borehole for all models.

Figure 3 depicts the model used for determining the effects of invasion. A borehole of radius r_m is shown penetrating a formation of true resistivity R_t. Invasion is shown to have taken place to a radius r_x with a resultant resistivity in the invaded zone of R_x.

The model used for determining the effects of bed thickness on the measured resistivity is shown in Figure 4. A bed of true resistivity R_t and having side beds of resistivity R_s is shown being penetrated by a borehole of radius r_m. The tool is assumed to be centered in the bed as shown, and the sidebeds are assumed to be infinitely thick. As in Figure 3, the borehole has mud of resistivity R_m; however, the effects of invasion are not considered here.

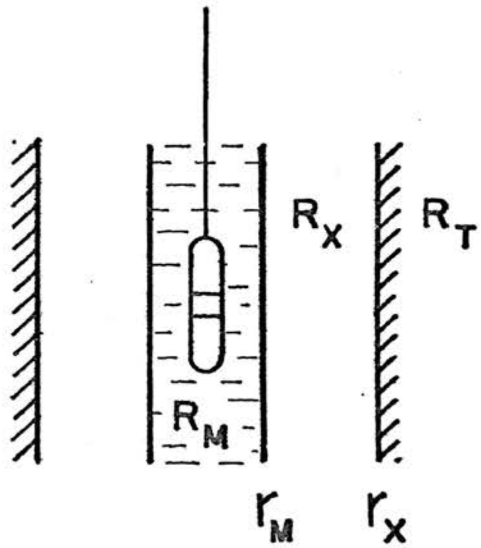


FIG. 3. MODEL FOR INVASION STUDIES.

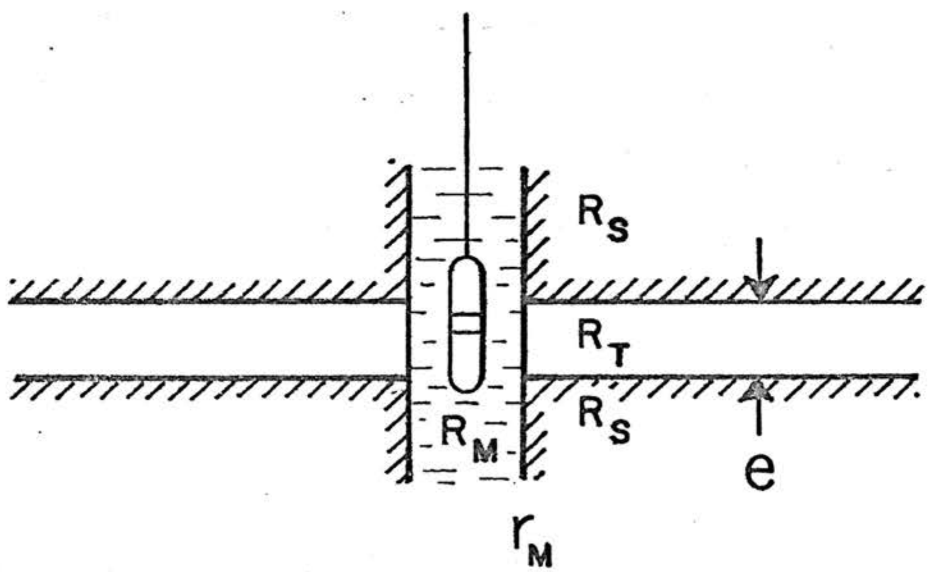


FIG. 4. MODEL FOR THIN BED STUDIES.

In either type of model, a cylindrical coordinate system is established with origin at the center of the measure electrode and the positive z-axis extending upward. The orientation of the x-axis is immaterial since there is complete symmetry about the z-axis. Note that for the models considered, symmetry also exists about the x-y plane and, hence, also about the origin. These symmetry conditions are not essential to the model, but they greatly reduce the computation required.

With the approach taken, before the apparent resistivity can be obtained, the potential distribution in the media surrounding the electrode must be determined. By using the potential gradients in the immediate vicinity of the measure electrode, the current flowing from the measure electrode can be determined. The apparent resistivity may then be obtained using Equation (1).

The potential distribution in the media surrounding the tool is determined by applying the condition for continuity of current. Equation (2) is the integral form expressing the condition for continuity of current where $\vec{J}$ is the current density.

$$\iint \vec{J} \cdot d\vec{\sigma} = 0 \tag{2}$$

This equation expresses the fact that for any region not enclosing any current sources or sinks, the integral of the current entering the region through all of its exterior surfaces is zero.

BOUNDARY CONDITIONS

If the total region considered around the tool is large enough, the remote boundary of this region may be assumed to be at zero potential.

The conducting surfaces of the tool itself are clamped at potential V.

The insulating surfaces have no outward current flow so the potential gradient normal to the surface of the insulator is zero. Therefore, next to these surfaces

$$\frac{\partial V}{\partial r} = 0 \tag{3}$$

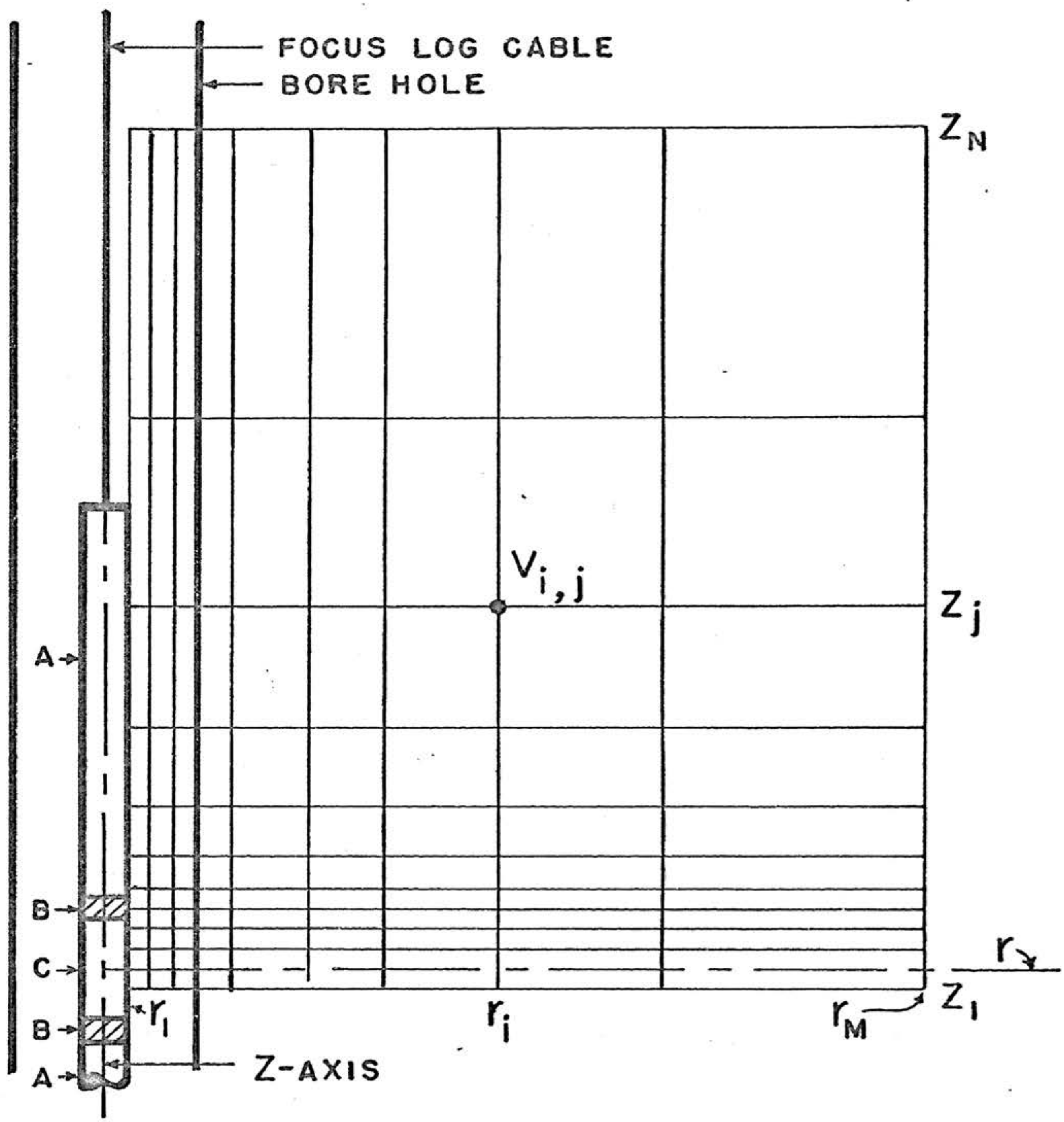
CHAPTER II

DISCRETIZATION OF THE MODEL

SPECIFICATION OF POTENTIAL DISTRIBUTION

The potentials are established at a fixed number of heights, $z_1, z_2, z_3 \cdots z_n$ from the tool, at distances $r_1, r_2, r_3, \cdots r_m$ from the axis of the electrode. Because of the symmetry about the axis of the electrode, this could be considered to be the potential distribution on a rectangular grid lying in any vertical plane, such as the x-z plane, which contains the axis of the electrode. The potentials need only be determined in one quadrant of this plane, since the potentials in the other quadrants may be obtained by symmetry.

In order to adequately describe the potential distribution near the electrodes, the mesh spacing must be made exceedingly small, and maintaining this spacing throughout the grid would result in a very large number of interior mesh points. Since the potential gradients are largest near the electrodes and decrease with increasing distance from the tool, one approach to decrease the number of interior mesh points is to increase the mesh spacing in some systematic manner with increasing distance from the tool. A diagrammatic representation of the first quadrant of such a mesh is shown in Figure 5.



A = GUARD ELECTRODES
B = INSULATORS
C = MEASURE ELECTRODE

FIG. 5. ONE QUADRANT OF THE TWO-DIMENSIONAL SPACE.

In the mesh chosen for the models studied, the spacing between successive grid lines is increased in a geometric progression. This method of spacing keeps the potential differences between grid lines approximately constant for a uniform media.

The placement of the individual grid lines is somewhat arbitrary as far as the validity of the model is concerned. The closeness of spacing in the mesh, however, ultimately determines the accuracy for the solution of the potential distribution. The following restrictions on the placement of the grid lines are imposed by the development of the model.

1. As shown in Figure 5, the boundaries between the conducting and insulating surfaces of the tool must lie midway between two grid lines.

2. Grid lines must be placed to coincide with the location of any boundaries between media of two differing resistivities. Thus, vertical lines must be placed at the surface of the tool, at the edge of the borehole, and at the edge of the invasion boundary. For the models of the type shown in Figure 4 for thin bed studies, horizontal grid lines must be placed to coincide with the location of the interfaces between the bed and its sidebeds.

The grid actually used in the studies of both types of models is shown in Figure 6. The aforementioned restrictions on the placement of grid lines affects only the lines passing near the tool. On the whole, the spacing may be observed to be increasing in a geometric progression with distance from the tool. A detail of the grid in the vicinity of the measure electrode is shown in Figure 7.

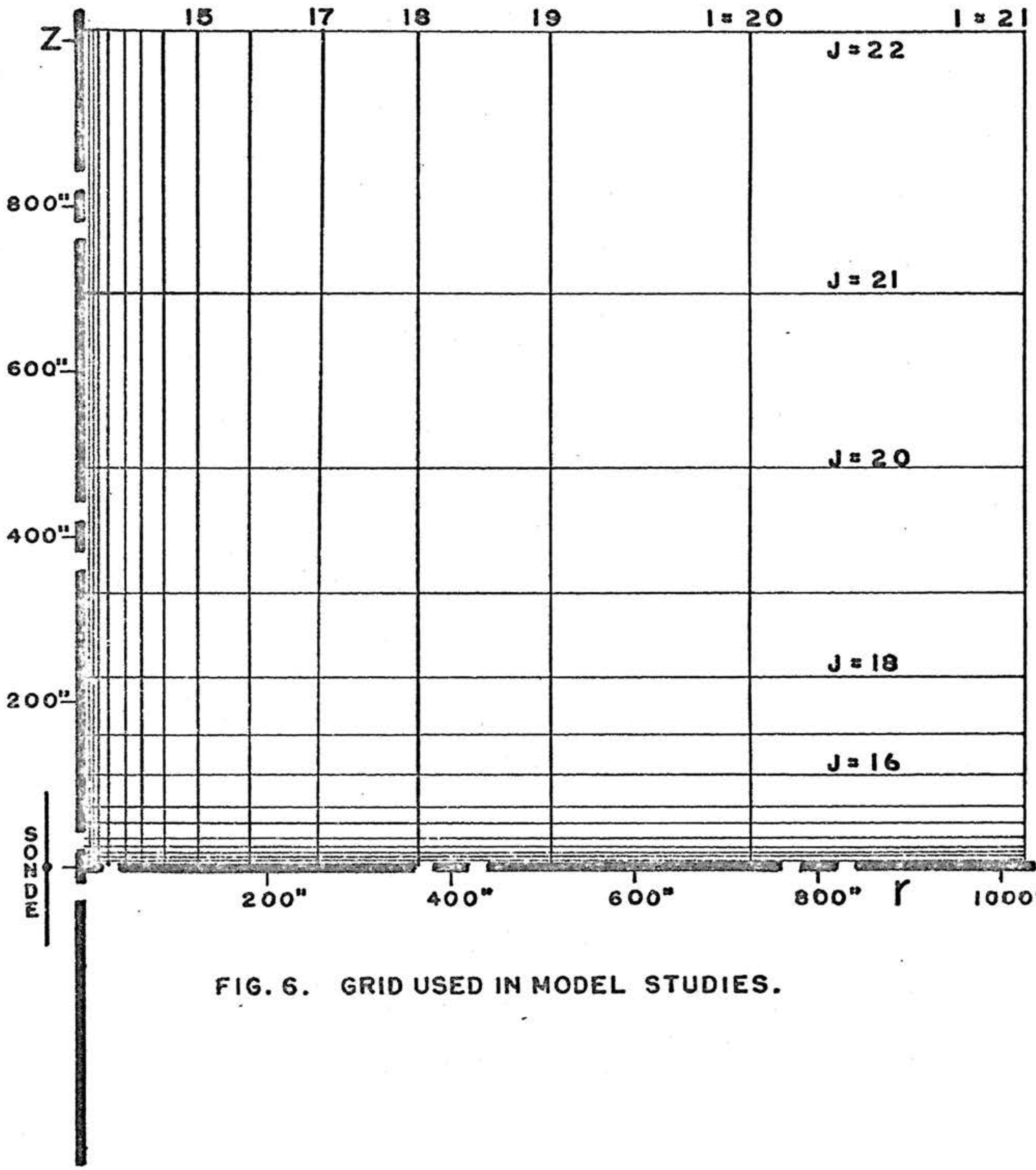


FIG. 6. GRID USED IN MODEL STUDIES.

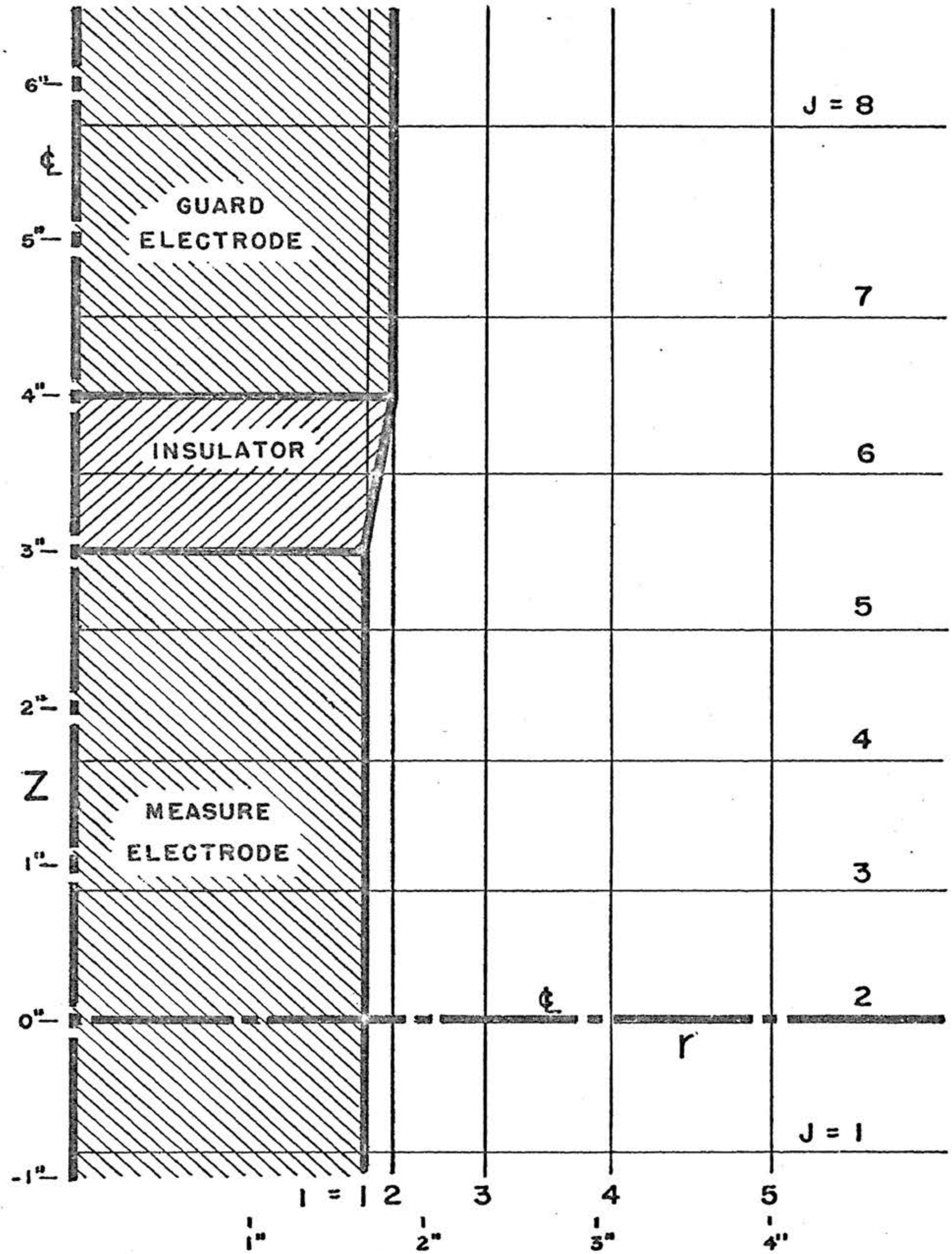


FIG. 7. DETAIL OF GRID IN VICINITY OF MEASURE ELECTRODE.

Let V_{ij} designate the potential at the intersection of the ith vertical grid line and the jth horizontal grid line. A value of 1 for i corresponds to the surface of the tool at its smallest dimension, and the farthest removed vertical grid line corresponds to i=M. The horizontal grid line passing through the center of the electrode is considered to be the second grid line, corresponding to j=2, with j=1 corresponding to a single grid line placed below the center of the electrode. This line is spaced below the centerline an amount equal to the spacing of the line corresponding to j=3 above the centerline. This single grid line from the fourth quadrant is required for symmetry in the calculations of the potential gradients, as will be shown later.

r_i is used to signify the distance from the axis of the electrode to the ith vertical grid line. z_j is the distance from the centerline of the electrode to the jth horizontal grid line. The values of r_i and z_j for the grid of Figures 6 and 7 may be found in Table 1.

Table 1

Values of r_i and z_j in inches for the grid shown in Figures 6 and 7.

i	r_i	j	z_j
1	1.687	1	-.833
2	1.812	2	.000
3	2.359	3	.833
4	3.072	4	1.667
5	4.000	5	2.500
6	5.657	6	3.500
7	8.000	7	4.500
8	11.314	8	5.734
9	18.000	9	8.295
10	22.627	10	12.000
11	32.000	11	17.359
12	45.255	12	25.112
13	64.000	13	36.327
14	90.510	14	52.550
15	128.000	15	76.019
16	181.020	16	109.970
17	256.000	17	159.080
18	362.040	18	230.130
19	512.000	19	332.900
20	724.080	20	481.580
21	1,024.000	21	696.650
		22	1,007.800

APPLICATION OF THE CONDITION FOR CONTINUITY OF CURRENT TO THE REGION SURROUNDING INTERIOR POINTS

Let (i,j) represent the point of intersection of the ith vertical grid line and the jth horizontal grid line.

Consider the toroidial region of rectangular x section constructed around any interior grid point (i,j) so that the boundaries of the region where this toroid intersects the plane of the grid lie midway between the point (i,j) and the adjacent grid lines, as shown in Figure 8. In general, it would be possible for regions of four differing resistivities R_1, R_2, R_3, and R_4 to meet at the mesh point (i,j) as shown in Figure 9. Since this region contains no current sources or sinks, the condition for continuity of current requires that the integral of the current flowing into the region over all of its surface be zero. To express this relation, the following symbols are introduced.

$I_{A_{i,j}}$: The component of the current flowing into the toroidal region normal to surface A of the toroid around point (i,j). As shown in Figure 9, surface A is the inside surface of the toroid.

$I_{B_{i,j}}$, $I_{C_{i,j}}$, $I_{D_{i,j}}$: Component of the current flowing into the toroidal region normal to surfaces B, C, and D, respectively, where surface B is the outside surface of the toroid, surface C is the bottom surface, and surface D is the top surface of the toroid.

I_{A_q}: Construct a rectangular coordinate system with origin at (i,j) and axes parallel to the grid lines.

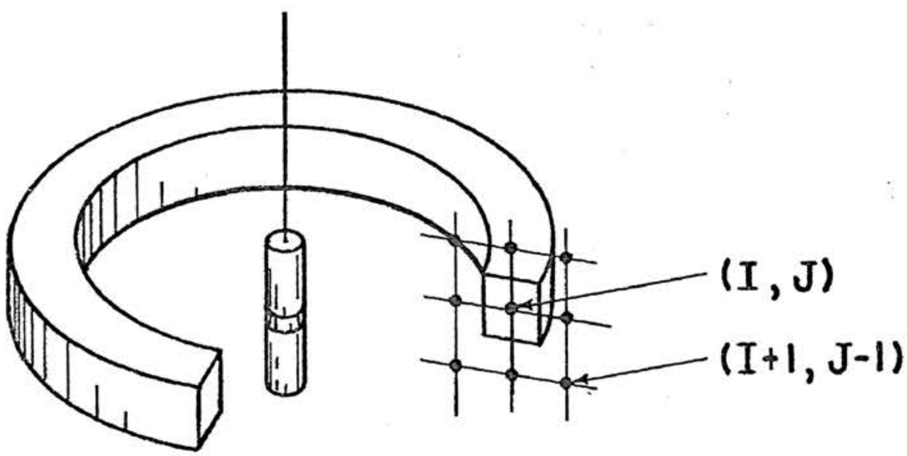


FIG. 8. TOROIDAL RING CONCENTRIC WITH THE ELECTRODE.

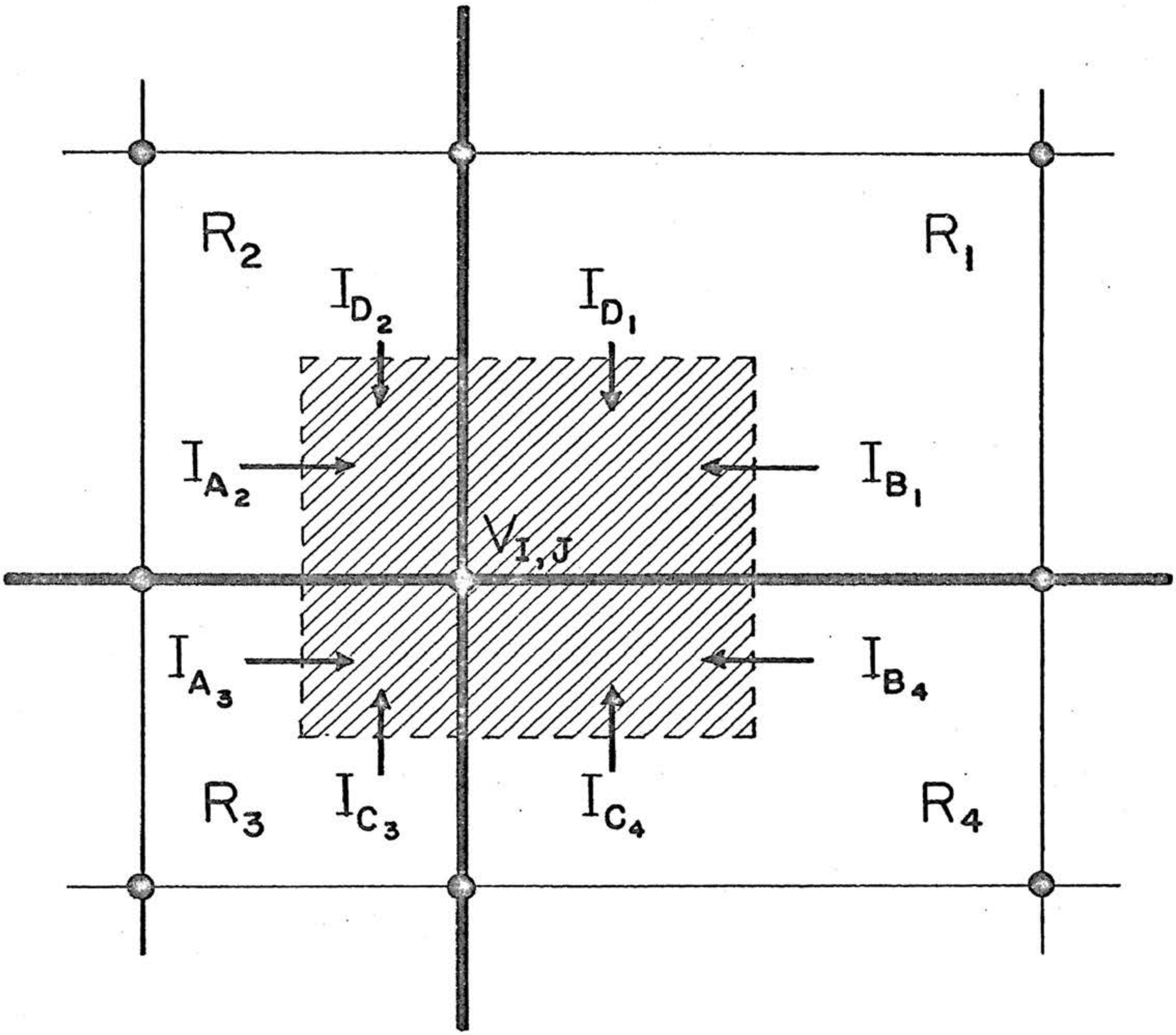


FIG. 9. RELATION OF TOROIDAL REGION TO GRID.

I_{A_q} represents that part of I_A which flows normal to surface A in quadrant q.

The condition for continuity of current in the toroidal region surrounding (i,j) requires

$$I_{A_{i,j}} + I_{B_{i,j}} + I_{C_{i,j}} + I_{D_{i,j}} = 0. \tag{4}$$

CALCULATION OF $I_{D_{i,j}}$ AND $I_{C_{i,j}}$

$I_{D_{i,j}}$ is found by calculating the current flow between the two horizontal surfaces shown in Figure 10, the lower surface at height z_j and potential $V_{i,j}$, and the upper surface at height z_{j+1} and potential $V_{i,j+1}$. Each of these two horizontal surfaces are divided into two parts. The current I_{D_2} flowing between the inner two surfaces passes through a region of resistivity R_2, and the current I_{D_1} flowing between the outer two surfaces passes through a region of resistivity R_1. $I_{D_{i,j}}$ is given by the relation

$$I_{D_{i,j}} = I_{D_1} + I_{D_2}. \tag{5}$$

The length of the current path for the current I_{D_2} is equal to $(z_{j+1}-z_j)$, and the current flows through a region of resistivity R_2

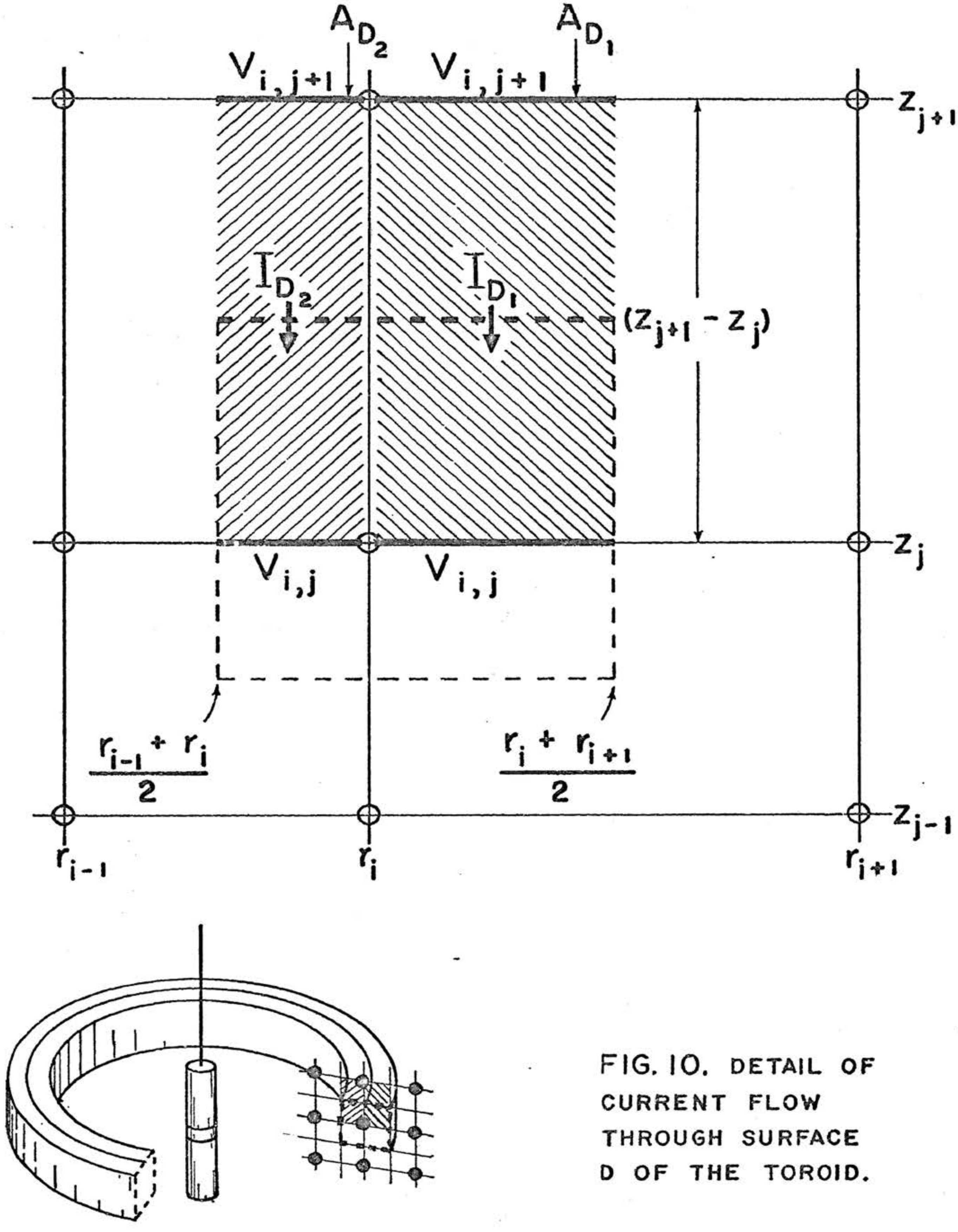


FIG. 10. DETAIL OF CURRENT FLOW THROUGH SURFACE D OF THE TOROID.

between surfaces of area

$$\pi\{r_i{}^2-[(r_{i-1}+r_i)/2]^2\}. \tag{6}$$

The current flow between these two surfaces is

$$I_{D_2} = [(\pi/R_2)\{r_i{}^2-[1/2(r_{i-1}+r_i)]^2\}/(z_{j+1}-z_j)] \cdot (V_{i,j+1}-V_{i,j}). \tag{7}$$

The length of the current path for the current I_{D_1} is equal to $z_{j+1}-z_j$, and the current flows through a region of resistivity R_1 between surfaces of area

$$\pi\{[1/2(r_{i+1}+r_i)]^2-r_i{}^2\} \tag{8}$$

The current flow between these two surfaces is then given by

$$I_{D_1} = [(\pi/R_1)\{[1/2(r_{i+1}+r_i)]^2-r_i{}^2\}/(z_{j+1}-z_j)] \cdot (V_{i,j+1}-V_{i,j}). \tag{9}$$

I_D is then given by substituting for I_{D_1} and I_{D_2} in Equation (5).

$$I_{D_{i,j}} = D'_{i,j}(V_{i,j+1}-V_{i,j}) \tag{10}$$

where

$$D'_{i,j} = (\pi/R_1)\{[1/2(r_{i+1}+r_i)]^2-r_i^2\}/(z_{j+1}-z_j)$$

$$+ (\pi/R_2)\{r_i^2-[1/2(r_{i-1}+r_i)]^2\}/(z_{j+1}-z_j). \qquad (11)$$

Simplifying (11),

$$D'_{i,j} = [\pi/(z_{j+1}-z_j)][(1/R_1)(r_{i+1}^2+2r_{i+}\ r_i-3r_i^2)$$

$$+(1/R_2)(3r_i^2-2r_{i-1}r_i-r_{i-1}^2)]. \qquad (12)$$

Similarly, $I_{C_{i,j}}$ may be computed:

$$I_{C_{i,j}} = I_{C_3}+ I_{C_4}, \qquad (13)$$

$$I_{C_3} = [(\pi/R_3)\{r_i^2-[1/2(r_{i-1}+r_i)]^2\}/(z_j-z_{j-1})]$$

$$\cdot(V_{i,j-1}-V_{i,j}), \qquad (14)$$

$$I_{C_4} = [(\pi/R_4)\{[1/2(r_{i+1}+r_i)]^2-r_i^2\}/(z_j-z_{j-1})]$$

$$\cdot(V_{i,j-1}-V_{i,j}). \qquad (15)$$

Substituting in (13),

$$I_{C_{i,j}} = C'_{i,j}(V_{i,j-1}-V_{i,j}), \tag{16}$$

where

$$C'_{i,j} = [\pi/(z_j-z_{j-1})][(1/R_3)(3r_i{}^2-2r_ir_{i-1}-r_{i-1}{}^2)$$

$$+(1/R_4)(r_{i+1}{}^2+2r_{i+1}r_i-3r_i{}^2). \tag{17}$$

CALCULATION OF I_A AND I_B

As shown in Figure 11, the length of the current path for both I_{A_2} and I_{A_3} is equal to $r_i - r_{i-1}$. The surface area, however, is not constant across this current path but varies as the current flows from the toroid's inner surface to its outer surface. The current flow between these surfaces must be obtained by integration.

$$I_{A_{i,j}} = I_{A_2} + I_{A_3} \tag{18}$$

Consider the voltage drop dV given in Equation (19) due to the current I_{A_2} flowing across the region of radius r and thickness dr as shown in Figure 12.

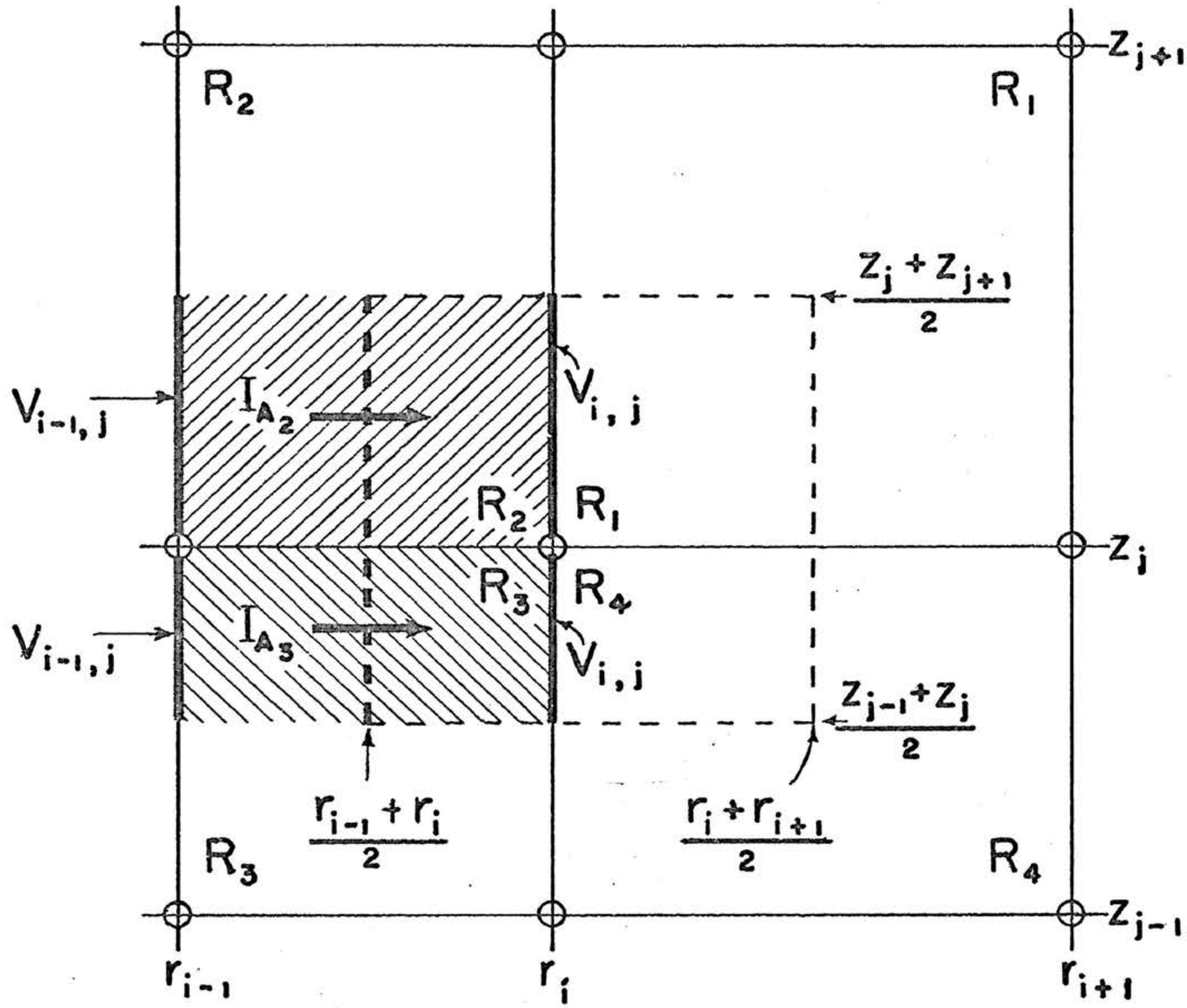


FIG. 11. DETAIL OF CURRENT FLOW THROUGH SURFACE A OF THE TOROID.

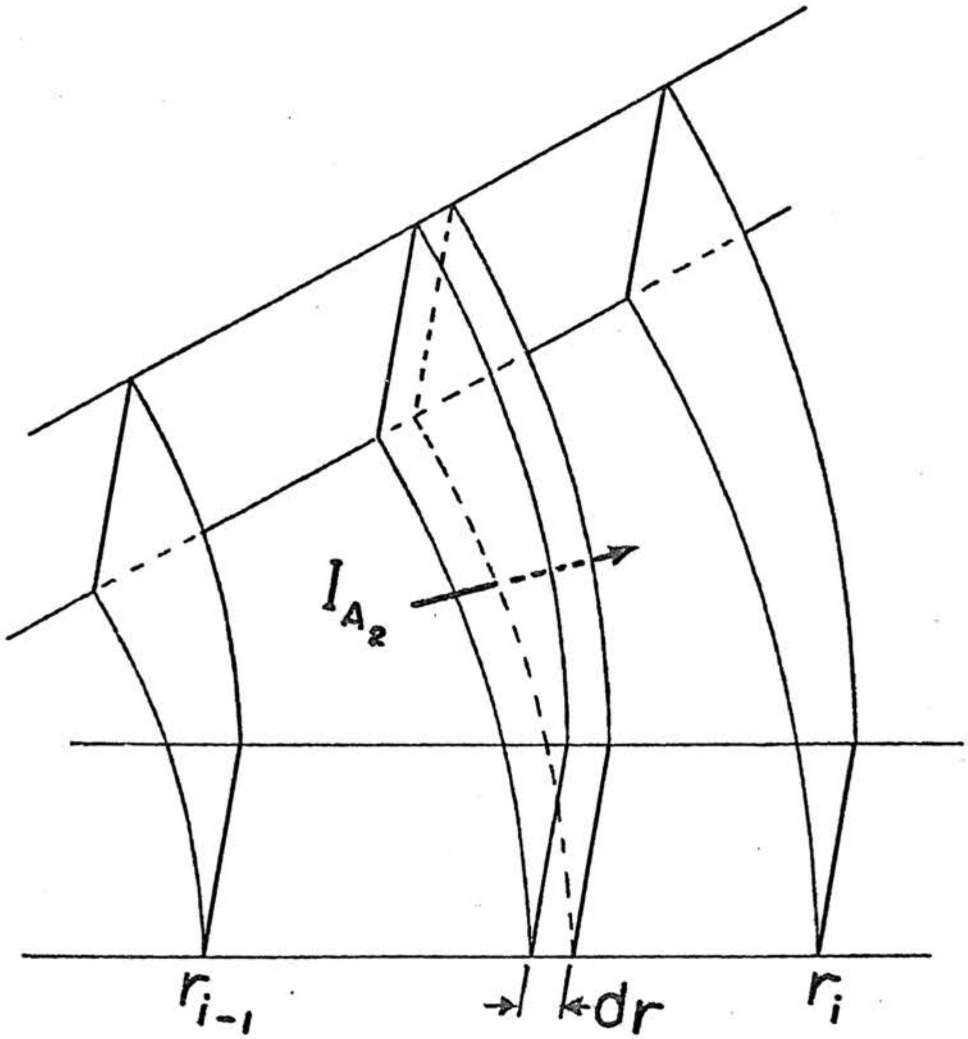


FIG. 12. INTEGRATION FOR CURRENT FLOW BETWEEN CYLINDRICAL SURFACES.

$$dV = \left[(-I_{A_2}R_2)/\{2\pi r[1/2(z_{j+1}+z_j)-z_j]\}\right]dr \tag{19}$$

The potential varies from $V_{i-1,j}$ to $V_{i,j}$ as one goes from r_{i-1} to r_i. Therefore,

$$\int_{V_{i-1,j}}^{V_{i,j}} dV = \{(-I_{A_2}R_2)/[\pi(z_{j+1}-z_j)]\} \int_{r_{i-1}}^{r_i} (1/r)dr \tag{20}$$

$$V_{i,j}-V_{i-1,j} = [(-I_{A_2}R_2)(\ln r_i - \ln r_{i-1})]/\pi(z_{j+1}-z_j) \tag{21}$$

$$I_{A_2} = \{\pi(z_{j+1}-z_j)/[R_2(\ln r_i - \ln r_{i-1})]\}(V_{i-1,j}-V_{i,j}) \tag{22}$$

Likewise, I_{A_3} may be obtained by integration.

$$I_{A_3} = \{\pi(z_{j+1}-z_j)/[R_3(\ln r_i - \ln r_{i-1})]\}(V_{i-1,j}-V_{i,j}) \tag{23}$$

Substituting for I_{A_2} and I_{A_3} in Equation (18)

$$I_{A_{i,j}} = A'_{i,j}(V_{i-1,j}-V_{i,j}); \tag{24}$$

where

$$A'_{i,j} = (\pi/R_2)(z_{j+1}-z_j)/(\ln r_i-\ln r_{i-1})$$

$$+(\pi/R_3)(z_j-z_{j-1})/(\ln r_i-\ln r_{i-1}). \qquad (25)$$

Simplifying (25)

$$A'_{i,j} = [\pi/(\ln r_i-\ln r_{i-1})][(z_{j+1}-z_j)/R_2$$

$$+(z_j-z_{j-1})/R_3]. \qquad (26)$$

Similarly $I_{B_{i,j}}$ may be determined.

$$I_{B_{i,j}} = I_{B_1}+ I_{B_4} \qquad (27)$$

$$I_{B_1} = \{\pi(z_{j+1}-z_j)/[R_1(\ln r_{i+1}-\ln r_i)]\}(V_{i+1,j}-V_{i,j}) \qquad (28)$$

$$I_{B_4} = \{\pi(z_j-z_{j-1})/[R_4(\ln r_{i+1}-\ln r_i)]\}(V_{i+1,j}-V_{i,j}) \qquad (29)$$

Substituting for I_{B_1} and I_{B_4} into Equation (27)

$$I_{B_{i,j}} = B'_{i,j}(V_{i+1,j}-V_{i,j}) \tag{30}$$

$$B'_{i,j} = [\pi/(\ln r_{i+1}-\ln r_i)][(z_{j+1}-z_j)/R_1+(z_j-z_{j-1})/R_2] \tag{31}$$

RELATION OF POTENTIAL AT AN INTERIOR POINT TO POTENTIALS AT ADJACENT GRID POINTS

The expressions for $I_{A_{i,j}}$, $I_{B_{i,j}}$, $I_{C_{i,j}}$ and $I_{D_{i,j}}$ given in Equations (24), (30), (16) and (10), respectively, may be substituted in Equation (4) yielding

$$A'_{i,j}(V_{i,j+1}-V_{i,j})+B'_{i,j}(V_{i,j-1}-V_{i,j})$$

$$+C'_{i,j}(V_{i-1,j}-V_{i,j})+D'_{i,j}(V_{i+1,j}-V_{i,j}) = 0 \tag{32}$$

Solving the above equation for $V_{i,j}$

$$V_{i,j} = A_{i,j}V_{i,j+1}+B_{i,j}V_{i,j-1}+C_{i,j}V_{i-1,j}+D_{i,j}V_{i+1,j} \tag{33}$$

for all i, j such that (i,j) is an interior point and where $A_{i,j}$, $B_{i,j}$, $C_{i,j}$ and $D_{i,j}$ are defined as follows:

$$A_{i,j} = (A'_{i,j})/(A'_{i,j}+B'_{i,j}+C'_{i,j}+D'_{i,j}), \tag{34}$$

$$B_{i,j} = (B'_{i,j})/(A'_{i,j}+B'_{i,j}+C'_{i,j}+D'_{i,j}), \tag{35}$$

$$C_{i,j} = (C'_{i,j})/(A'_{i,j}+B'_{i,j}+C'_{i,j}+D'_{i,j}), \tag{36}$$

$$D_{i,j} = (D'_{i,j})/(A'_{i,j}+B'_{i,j}+C'_{i,j}+D'_{i,j}). \tag{37}$$

Equation (33) relates the value of the potential at the interior grid point (i,j) to the potentials at surrounding grid points. One equation of the above form may be written for each interior point of the grid.

It may be noted that the above equation shows that the potential at any interior point on the mesh is simply a weighted average of the potentials at the four surrounding mesh points, with the weights, given by $A_{i,j}$, $B_{i,j}$, $C_{i,j}$ and $D_{i,j}$, being functions of the mesh spacings and of the resistivities of the media immediately surrounding the given mesh point.

For the grid of Figure 6, there are 344 interior points. The 344 equations given by (33) together with the boundary conditions comprise a set of simultaneous equations, the solution of which yields the potential distribution for the grid.

The solution of these equations for the grid of Figure 6 is discussed in the following chapter.

SIMPLE INTERIOR POINTS

A simple interior point is an interior point which is surrounded by a media of resistivity R, such that $R_1=R_2=R_3=R_4=R$.

For simple interior points the coefficients $A_{i,j}$, $B_{i,j}$, $C_{i,j}$ and $D_{i,j}$ in Equation (33) may be simplified to the following forms.

$$A_{i,j} = A''_{i,j}/E_{i,j} \tag{38}$$

$$B_{i,j} = B''_{i,j}/E_{i,j} \tag{39}$$

$$C_{i,j} = C''_{i,j}/E_{i,j} \tag{40}$$

$$D_{i,j} = D''_{i,j}/E_{i,j} \tag{41}$$

where

$$E_{i,j} = A''_{i,j}+B''_{i,j}+C''_{i,j}+D''_{i,j} \tag{42}$$

$$A''_{i,j} = A'_{i,j}(R) = \pi(z_{j+1}-z_{j-1})/(\ln r_i-\ln r_{i-1}) \tag{43}$$

$$B''_{i,j} = B'_{i,j}(R) = \pi(z_{j+1}-z_{j-1})/(\ln r_{i+1}-\ln r_i) \tag{44}$$

$$C''_{i,j} = C'_{i,j}(R)$$

$$= \pi(r_{i+1}{}^2+2r_{i+1}r_i-2r_ir_{i-1}-r_{i-1}{}^2)/(z_i-z_{i-1}) \tag{45}$$

$$D''_{i,j} = D'_{i,j}(R)$$

$$= \pi(r_{i+1}{}^2+2r_{i+1}r_i-2r_ir_{i-1}-r_{i-1}{}^2)/(z_{j+1}-z_j) \tag{46}$$

The coefficients $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, and $D_{i,j}$ may be observed to be independent of the resistivity R for simple interior points. Thus, the coefficients in Equation (33) for interior points not lying on resistance boundaries are invariant from model to model.

NUMERICAL REPRESENTATION OF BOUNDARY CONDITIONS

REMOTE BOUNDARIES

Along all the remote boundaries the potential is assumed to be zero. For the grid of Figure 6, this condition is represented by the following equations.

$$V_{i,22} = 0.0 \qquad i = 3,4,5,\cdots,21 \tag{47}$$

$$V_{21,j} = 0 \qquad j = 1,2,3,\cdots,22 \tag{48}$$

MEASURE ELECTRODE SURFACE

The surface of the measure electrode is maintained at a potential of one volt. For the grid of Figure 6

$$V_{1,j} = 1.0 \qquad j = 1,2,3,4,5 \tag{49}$$

GUARD ELECTRODE SURFACE

The surface of the guard electrode is maintained at one volt. For the grid of Figure 6

$$V_{1,j} = 1.0 \qquad j = 7,8,9,\cdots,15 \tag{50}$$

INSULATING SURFACES OF THE TOOL

As discussed previously, the potential gradient normal to the insulating surfaces of the tool is zero. For the grid of Figure 6 this is expressed by the equations:

$$V_{2,6} = V_{3,6} \tag{51}$$

$$V_{2,j} = V_{3,j} \qquad j = 16,17,18,\cdots,21 \tag{52}$$

PLANE OF SYMMETRY

The condition of symmetry about the plane z=0 allows the potentials to be calculated in only one quadrant of the mesh. This condition is imposed on the potentials by setting the potentials in the one line from the fourth quadrant equal to the corresponding line of potentials in the first quadrant. For the grid of Figure 6

$$V_{i,1} = V_{i,3} \ . \tag{53}$$

The irregular shaped boundary thus described encloses the 344 interior points whose potentials satisfys the relation given in Equation (33).

CHAPTER III

NUMERICAL SOLUTION FOR THE POTENTIAL DISTRIBUTION

The solution of the simultaneous equations described by Equation (33) yields the potential distribution across the mesh. For a mesh the size of Figure 6, the storage of all the coefficients for the 344 simultaneous equations would require $344 \times 344 = 118{,}336$ locations, precluding the use of a direct method of solution.

An iterative method, the method of successive displacements (the Gauss-Seidel method) is used, requiring the storage of only the nonzero coefficients $A_{i,j}$, $B_{i,j}$, $C_{i,j}$ and $D_{i,j}$, for each interior point.

The iterative solution of these equations is not without difficulty, however, as an initial trial solution is required and convergence is so slow that some form of acceleration must be employed.

ITERATIVE SOLUTION OF THE SIMULTANEOUS EQUATIONS

For each interior point (i,j) in turn, Equation (33) is solved for $V_{i,j}$ using the most recent values for the potentials at the surrounding points. Let $V_{i,j}^{(k)}$ denote the value of the potential $V_{i,j}$ computed at the end of the kth iteration. Denoting the newly computed value of $V_{i,j}$ by $V_{i,j}^{(k+1/2)}$, we have, from Equation (33),

$$V_{i,j}^{(k+1/2)} = A_{i,j}V_{i,j}^{(k)} + B_{i,j}V_{i,j-1} + C_{i,j}V_{i+1,j} + D_{i,j}V_{i+1,j} \tag{54}$$

The value for $V_{i,j}^{(k+1)}$, representing the final value of V for the (k+1) iteration is then given by

$$V_{i,j}^{(k+1)} = V_{i,j}^{(k)} + F(V_{i,j}^{(k+1/2)} - V_{i,j}^{(k)}) \qquad 1 \le F < 2 \tag{55}$$

Equation (55) is employed as a device to accelerate convergence and is known as overrelaxation. The choice of F as 1.0 is equivalent to using no overrelaxation, and Equation (55) reduces to

$$V_{i,j}^{(k+1)} = V_{i,j}^{(k+1/2)} \tag{56}$$

Equations (54) and (55) are employed for each interior point in turn, and the new value obtained for $V_{i,j}$ is stored immediately and used in all further computations involving the potential at that point. For that reason, the superscripts were not used for the last three potentials involved in Equation (44), as some of the terms involve potentials calculated in the kth iteration and some involve potentials from the (k+1) iteration.

It was found that the iterations progress more smoothly with a more consistent trend to the final potential distribution if the interior points on the mesh are swept periodically in a different order. The points of the grid are swept first in the order shown in Figure 13(a), then as shown in Figure 13(b) on the next iteration, the next iteration as shown in Figure 13(c), then as shown in Figure 13(d). The order of Figure 13(a) is then used and the sequence is repeated cyclically.

The inverse of the current flowing from the measure electrode, R_a', is proportional to the apparent resistivity of the medium. This quantity is computed after each iteration, and the convergence of this quantity is used to judge the convergence of the iterations.

As shown previously, for simple interior points the terms of Equation (33) are independent of the resistivity. For the model where the entire media surrounding the tool is of uniform resistivity, all interior points are simple interior points, and hence the potential distribution is independent of the resistivity. This distribution is derived and used for initial values, $V_{i,j}^{(0)}$, in all other model studies.

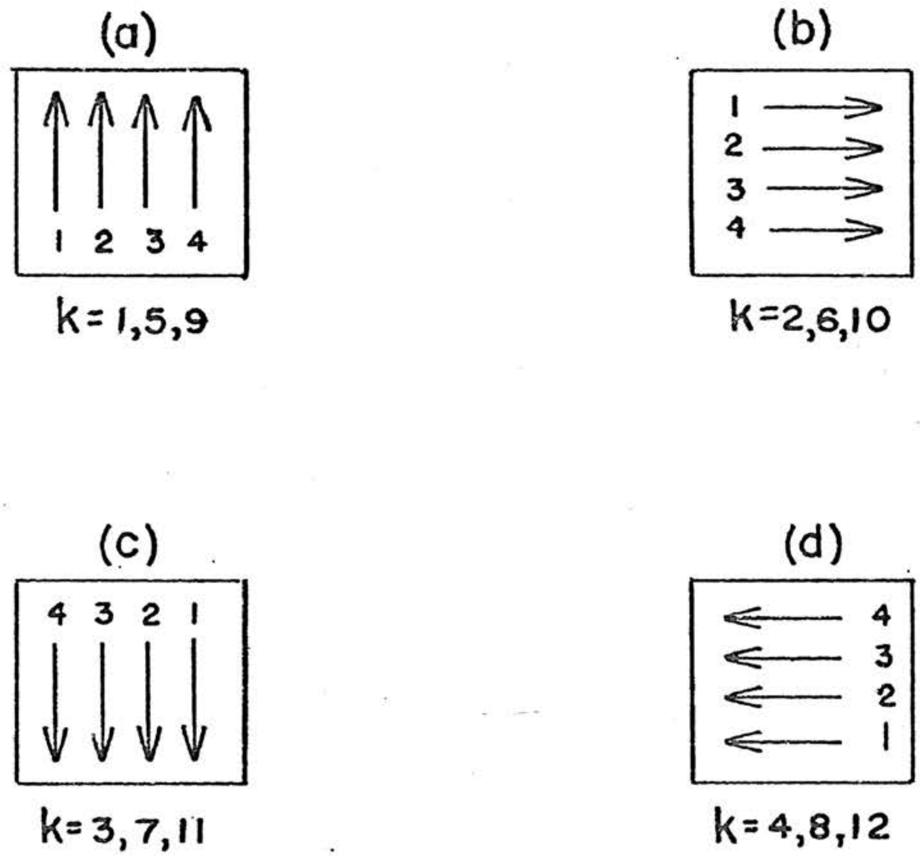


FIG.13. ALTERNATING DIRECTION SWEEP OF INTERIOR POINTS.

OBTAINING THE TRIAL SOLUTION

The potential distribution obtained for the case where the entire media surrounding the tool is of uniform resistivity is used as the source of initial values, $V_{i,j}^{(o)}$, in all other model studies.

This distribution is obtained by initially setting the potentials along the conducting surfaces of the tool to one volt and the potentials at all other grid points to zero. As the media is assumed to be entirely uniform, the apparent resistivity should equal the true resistivity of the medium. The constant of proportionality between the calculated apparent resistivity and the apparent resistivity may be determined from this model. Iterations are performed until the calculated apparent resistivity indicates convergence to the desired accuracy.

The Appendix contains the potential distribution obtained after every 400 iterations for this model using a near optimal value of 1.8 for the overrelaxation factor. A resistivity of one ohm-meter was assumed for the apparent resistivity calculations. After 8000 iterations R_a' had attained a value of 4.0297961, differing by less than $1.\times10^{-8}$ between successive iterations. Thus it follows that:

$$R_a = R_a'/4.0297961 \tag{57}$$

A contour map of the final potential distribution is given in Figure 13. It should be noted that Figure 13 is a log-log plot, distorting the small measure electrode to an infinite size.

The rate of convergence is sensitive to the choice of the overrelaxation factor. The convergence of the calculated apparent resistivity is shown in Figure 14 for the uniform media of resistivity one ohm-meter. As an example, the solution for R_a' is accurate within 5% $(2.\times10^{-1})$ after only 640 iterations with F=1.8, whereas 2000 iterations were required to attain the same accuracy with F=1.0.

COMPUTATIONAL METHOD, GENERAL

The coefficients of Equation (33) have been shown to be invariant from model to model with the exception of those occurring on resistance boundaries. These coefficients are therefore computed first for the uniform medium case and saved. Those coefficients for interior points located on resistance boundaries are recomputed and replaced by the original values at the conclusion of the run for that model.

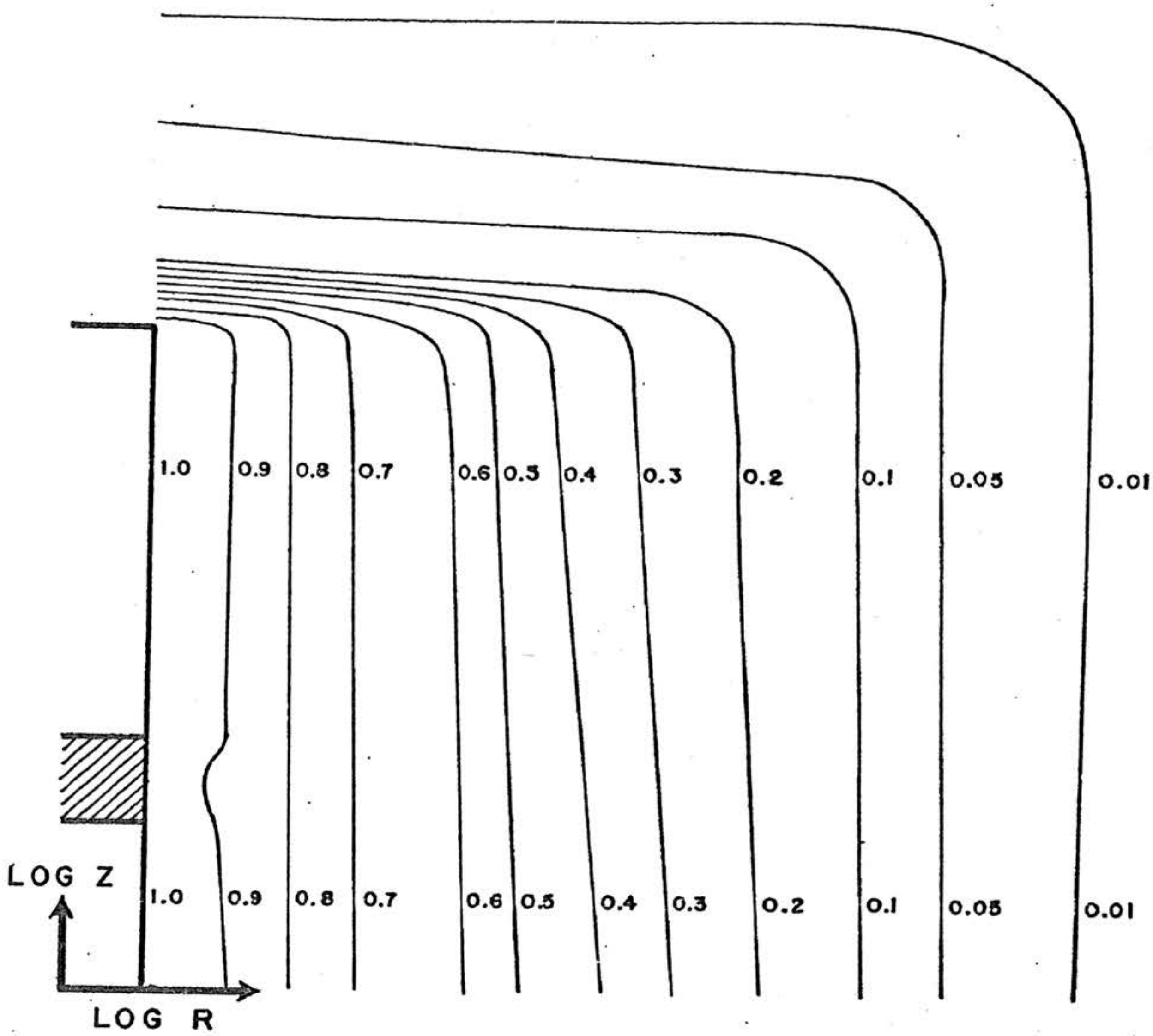


FIG. 14. POTENTIAL DISTRIBUTION FOR UNIFORM MEDIA.

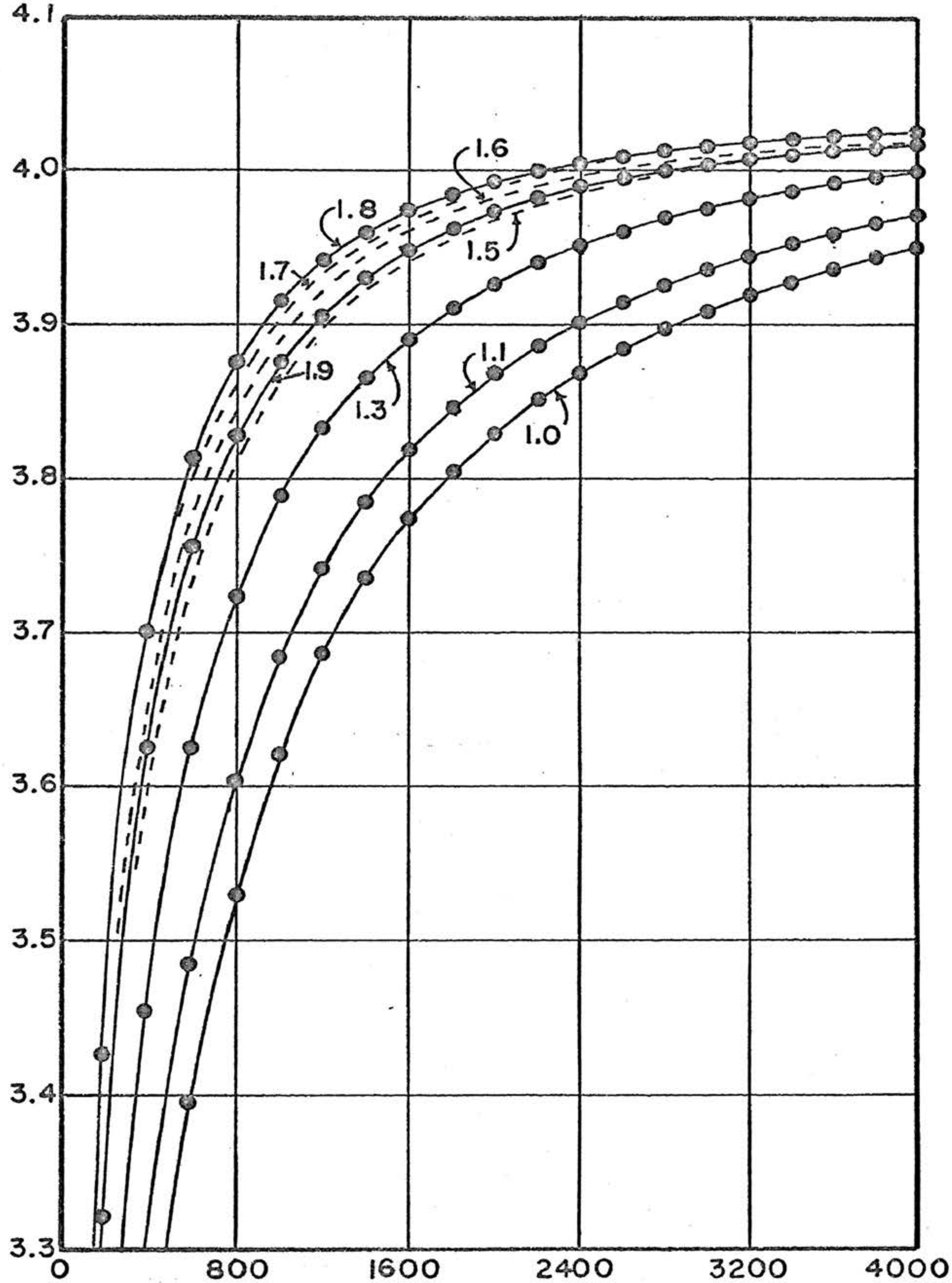


FIG. 15. INFLUENCE OF OVER-RELAXATION FACTOR ON RATE OF CONVERGENCE.

CHAPTER IV

RESULTS AND CONCLUSIONS

The two models shown in Figures 3 and 4 were programmed on the Univac 1108 and on the CDC 3600 high speed digital computers. Selected results from these runs are presented here.

INVASION STUDIES

The model used for invasion studies is shown in Figure 3. The coefficients $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, and $D_{i,j}$ are first computed for the uniform media. For each case considered, the coefficients for the interior points lying on the borehole boundary are recomputed using Equations (34) through (37) and Equations (11), (17), (26), and (31). These coefficients are evaluated with $R_{1_{i,j}}=R_{4_{i,j}}=R_t$ and $R_{2_{i,j}}=R_{3_{i,j}}=R_x$.

Iterations are then performed until R_a' converges to within the desired accuracy. At the termination of the run the original coefficients are restored along the two boundaries.

The results obtained for three different values of R_m, R_x, and R_t are shown graphically in Figures 16, 17, and 18. The value of apparent resistivity is shown plotted against invasion depth.

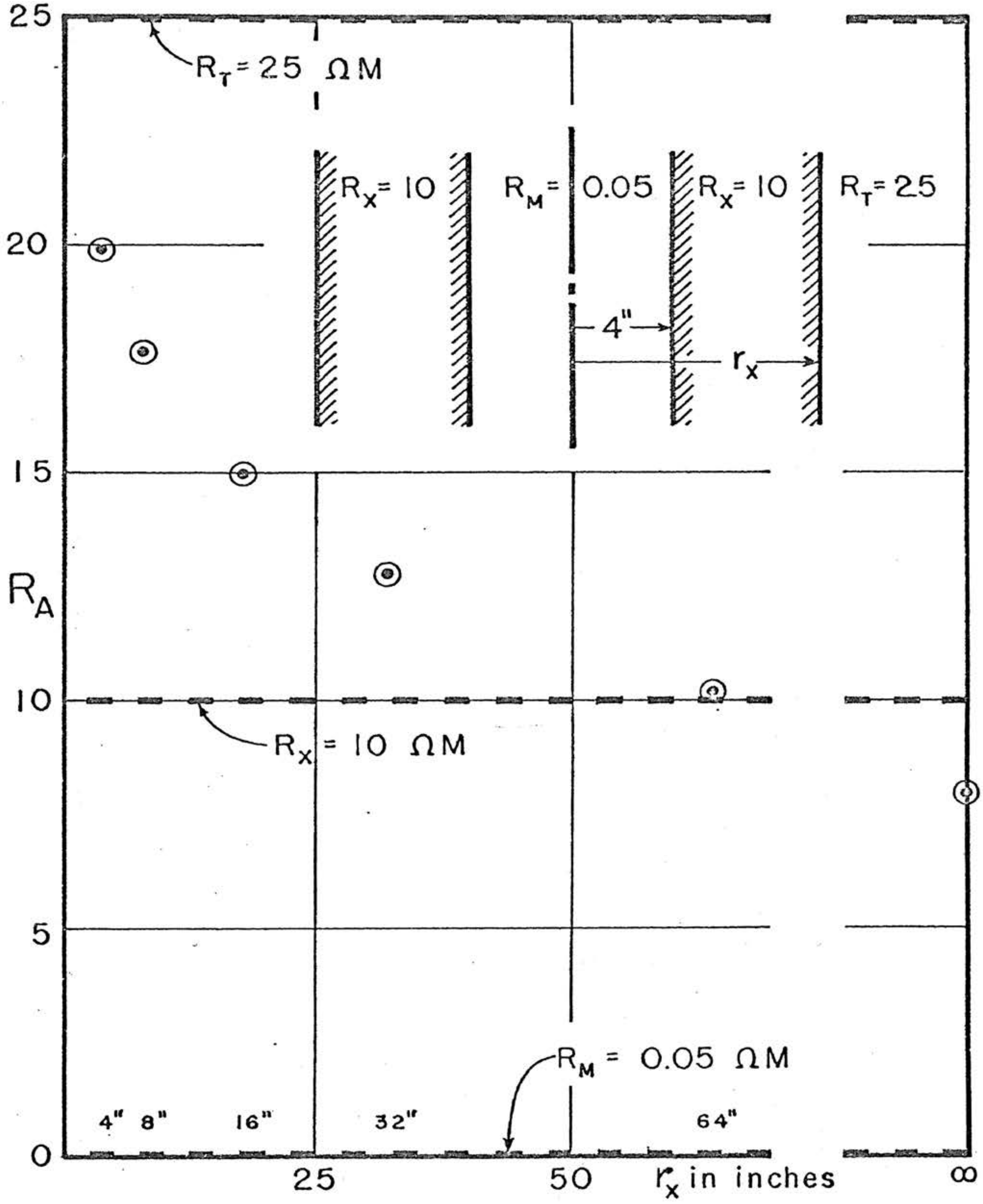


FIG. 16. INVASION STUDY, CASE I.

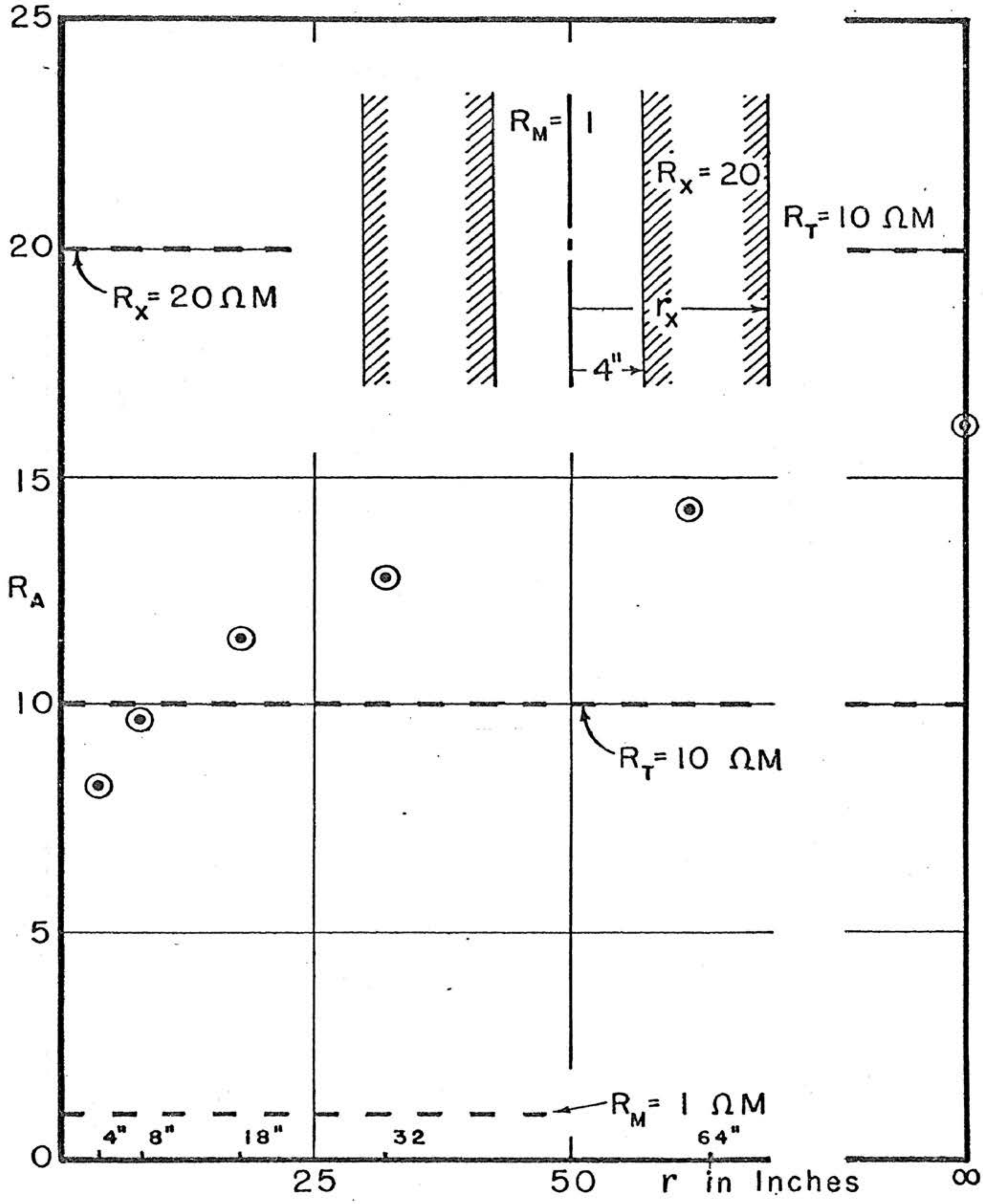


FIG. 17. INVASION STUDY, CASE 2.

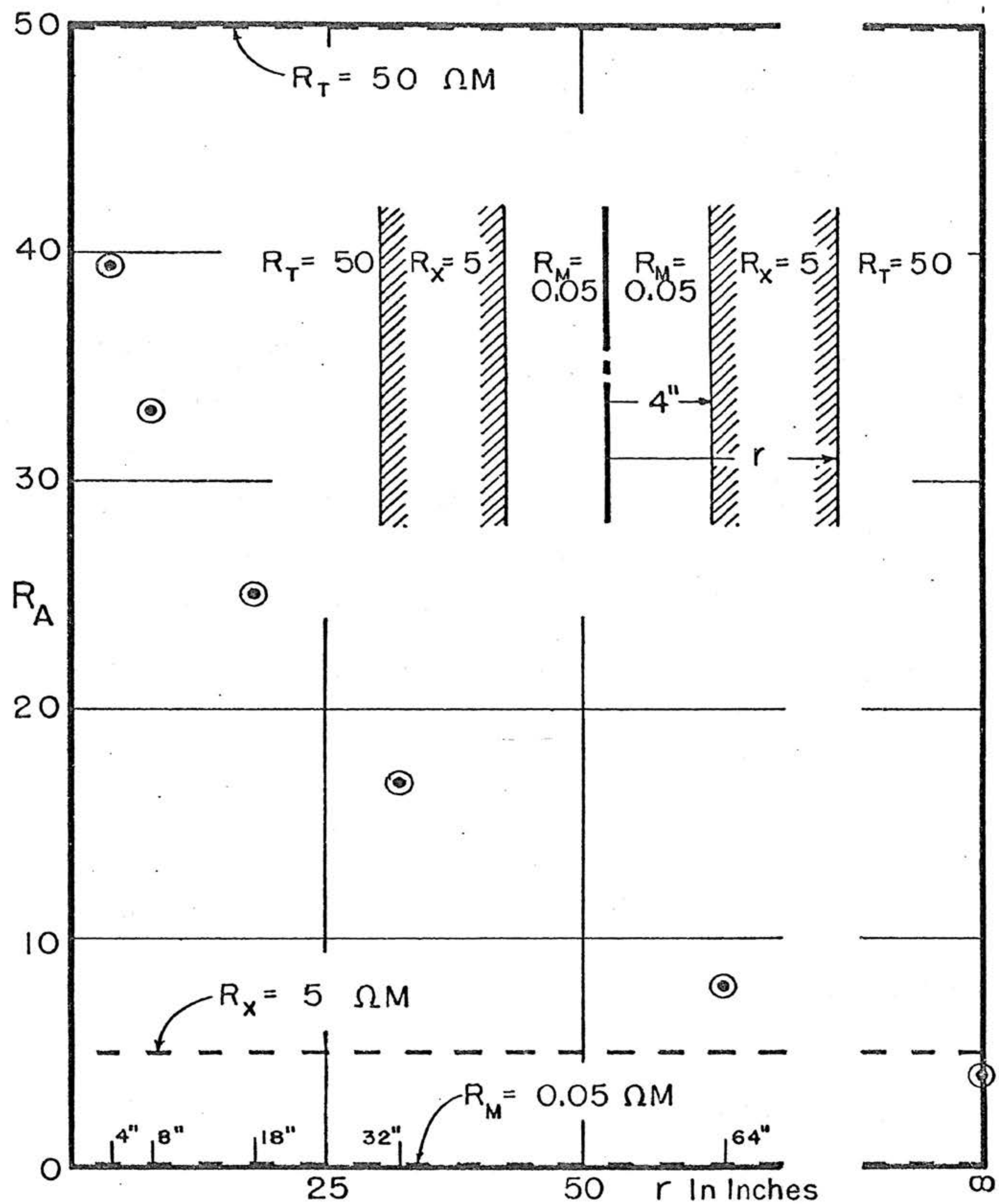


FIG. 18. INVASION STUDY, CASE 3.

THIN BED STUDIES

The model used for thin bed studies is shown in Figure 4. The coefficients are recomputed along the resistance boundaries as in the previous examples. These boundaries are the borehole boundary, the sidebed boundary, and the corner point, which is the point of intersection of the previous two boundaries.

The coefficients are recomputed at the corner point using $R_1=R_s$, $R_2=R_3=R_m$, and $R_4=R_t$. Those along the sidebed boundary are computed with $R_1=R_2=R_s$, and $R_3=R_4=R_t$. The borehole boundary is divided into two parts. Along that part of the boundary next to the sidebeds $R_1=R_4=R_s$, $R_2=R_3=R_m$. Next to the center bed $R_1=R_4=R_t$ and $R_2=R_3=R_m$.

Results from this study are presented graphically in Figures 19, 20, and 21, plotting apparent resistivity versus bed thickness for three different resistivity configurations.

It may be noted that there is an inflection in the curve, as might be expected, when the bed thickness approaches the dimension of the tool.

SUMMARY

The mathematical model presented here for the focus well logging tool is applicable to any well logging tool of an electrode type. Resistance boundaries either cylindrical about the borehole or horizontal in form may be considered in any combination.

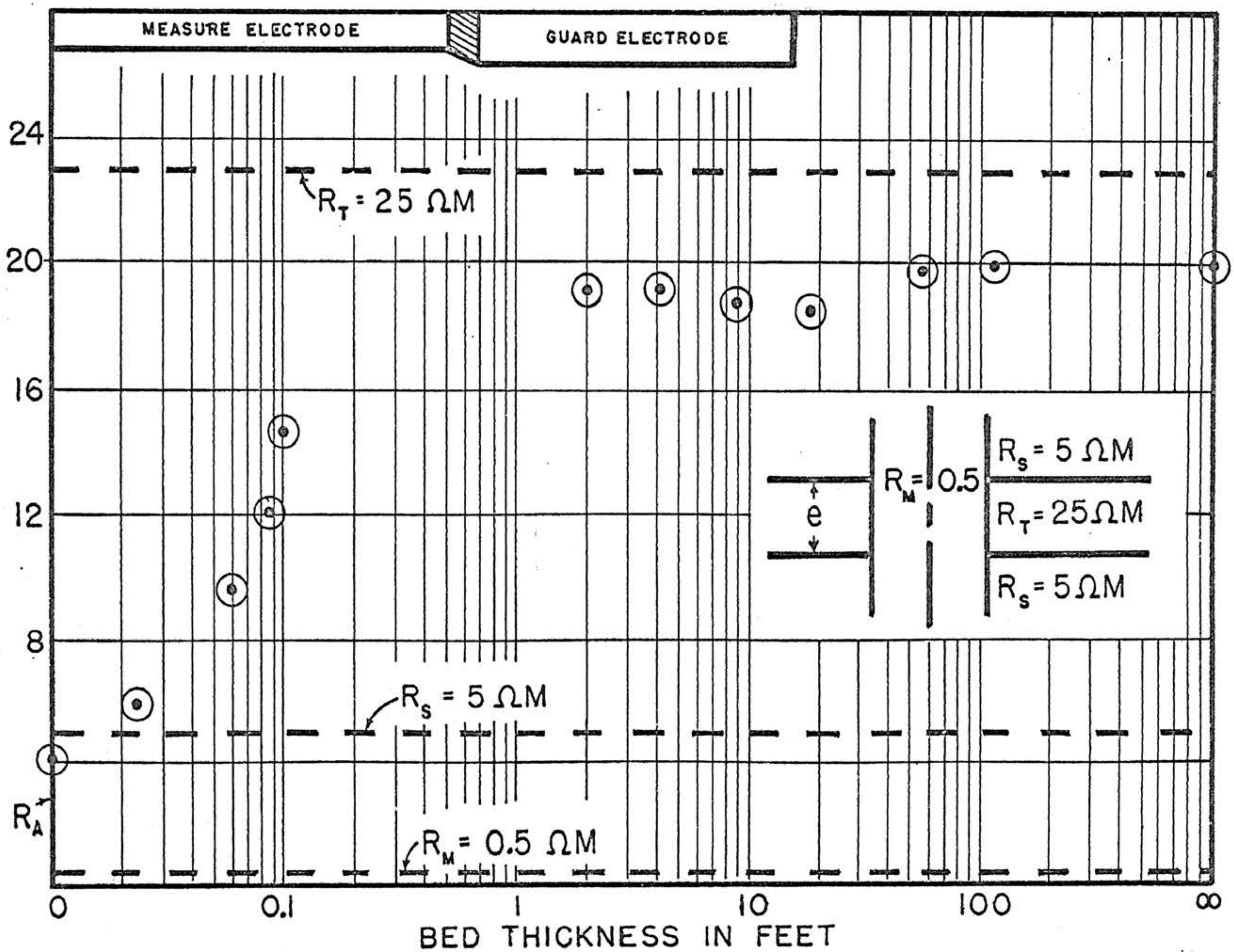


FIG. 19. THIN BED STUDY, CASE I.

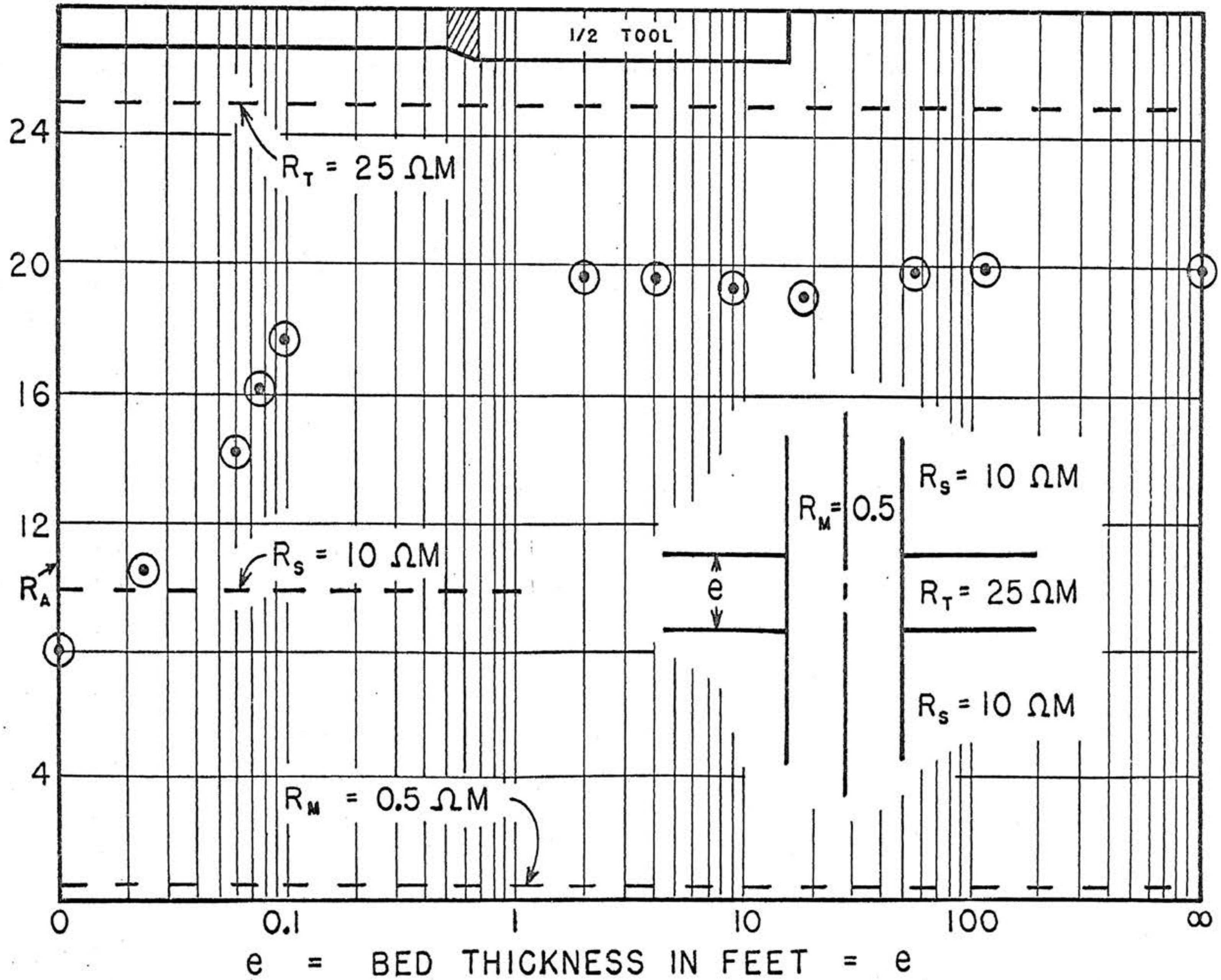
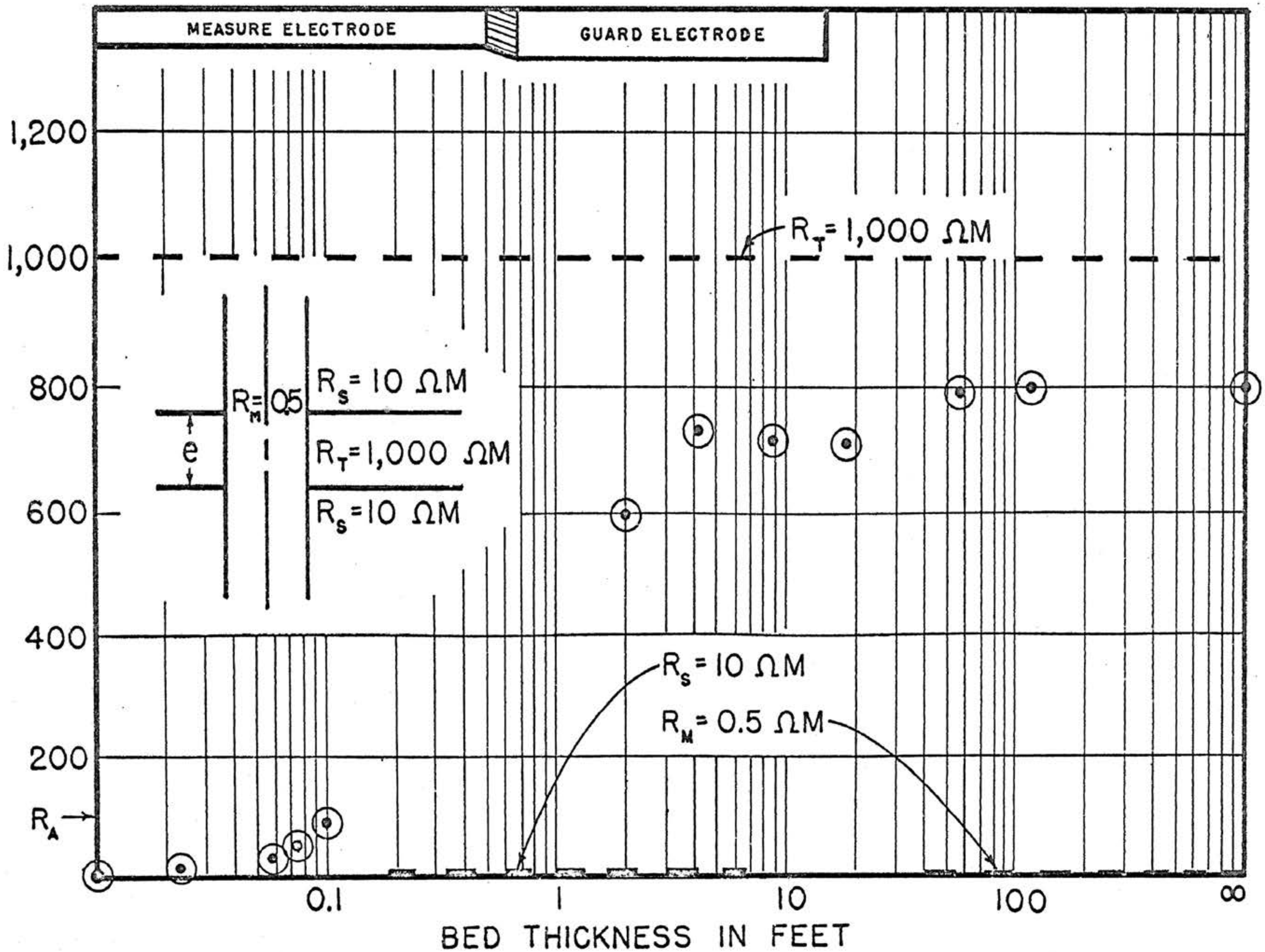


FIG. 20. THIN BED STUDY, CASE 2.


MEASURE ELECTRODE
GUARD ELECTRODE
1,200
1,000
800
600
400
200
R_A
R_T = 1,000 ΩM
R_M = 0.5
R_S = 10 ΩM
e
R_T = 1,000 ΩM
R_S = 10 ΩM
R_S = 10 ΩM
R_M = 0.5 ΩM
0.1
1
10
100
∞
BED THICKNESS IN FEET

The two models presented here possess symmetry about a horizontal plane through the center of the tool. Though it greatly reduces the required computations, this symmetry is not required for the validity of the model, symmetry about the axis of the tool being the only requirement.

The model is discretized by compartmentalizing the tool and its environment. The integral form of the condition for continuity of current is then applied to each compartment, yielding a very large system of equations.

The iterative solution of these equations, required because of the extreme storage and time requirements of a direct method, is very slowly convergent, requiring the use of an acceleration technique such as overrelaxation.

With these methods, the numerical solution for the current flow from the measure electrode is now economically feasible on present-day high-speed digital computers. A run for a model of the type presented will take under 15 seconds for convergence to within 1% on a machine such as the Univac 1108.

The method is flexible in that it will handle a large number of different types of resistance boundaries, the only major requirement being symmetry about the axis of the borehole tool.

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APPENDIX

POTENTIAL DISTRIBUTION

The following pages are printouts from the Univac 1108 Fortran IV program used to obtain the potential distribution for the uniform media of resistivity one ohm-meter.

The printouts are of the potential distribution obtained for the grid of Figure 6 after each 800 iterations. An over-relaxation factor of 1.8 was used.

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0071	.0071	.0071	.0071	.0071	.0071	.0071	.0071	.0071	.0071	.0071
J= 20	.0171	.0171	.0171	.0171	.0171	.0171	.0171	.0171	.0172	.0171	.0171
J= 19	.0335	.0335	.0335	.0335	.0335	.0335	.0336	.0336	.0336	.0335	.0334
J= 18	.0616	.0616	.0616	.0616	.0616	.0616	.0616	.0616	.0614	.0612	.0606
J= 17	.1125	.1125	.1125	.1125	.1125	.1125	.1123	.1120	.1108	.1097	.1070
J= 16	.2274	.2274	.2274	.2272	.2268	.2256	.2233	.2192	.2091	.2018	.1867
J= 15	1.0000	1.0000	.9298	.8597	.7902	.6998	.6112	.5254	.4175	.3676	.2982
J= 14	1.0000	1.0000	.9378	.8755	.8133	.7317	.6502	.5691	.4622	.4106	.3355
J= 13	1.0000	1.0000	.9396	.8792	.8187	.7394	.6601	.5811	.4759	.4247	.3490
J= 12	1.0000	1.0000	.9402	.8804	.8205	.7420	.6635	.5852	.4809	.4299	.3542
J= 11	1.0000	1.0000	.9404	.8807	.8210	.7426	.6644	.5864	.4826	.4318	.3563
J= 10	1.0000	1.0000	.9402	.8804	.8206	.7422	.6641	.5864	.4830	.4325	.3570
J= 9	1.0000	1.0000	.9397	.8793	.8192	.7408	.6631	.5859	.4830	.4326	.3572
J= 8	1.0000	1.0000	.9376	.8760	.8159	.7385	.6619	.5854	.4828	.4326	.3571
J= 7	1.0000	1.0000	.9307	.8699	.8122	.7370	.6612	.5851	.4827	.4325	.3571
J= 6	.9239	.9239	.9014	.8598	.8089	.7359	.6608	.5849	.4827	.4325	.3570
J= 5	1.0000	.9807	.9195	.8637	.8088	.7354	.6605	.5848	.4826	.4324	.3570
J= 4	1.0000	.9837	.9240	.8660	.8093	.7352	.6603	.5847	.4826	.4324	.3570
J= 3	1.0000	.9841	.9253	.8671	.8097	.7351	.6602	.5847	.4826	.4324	.3570
J= 2	1.0000	.9841	.9256	.8674	.8099	.7351	.6602	.5847	.4826	.4324	.3570
J= 1	1.0000	.9841	.9253	.8671	.8097	.7351	.6602	.5847	.4826	.4324	.3570

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0071	.0071	.0070	.0068	.0065	.0059	.0049	.0034	.0017	.0000
J= 20	.0171	.0169	.0166	.0159	.0148	.0128	.0099	.0063	.0028	.0000
J= 19	.0331	.0325	.0314	.0293	.0258	.0207	.0144	.0082	.0033	.0000
J= 18	.0595	.0573	.0534	.0473	.0385	.0279	.0174	.0089	.0034	.0000
J= 17	.1022	.0943	.0827	.0674	.0499	.0329	.0187	.0089	.0032	.0000
J= 16	.1664	.1414	.1132	.0844	.0576	.0352	.0189	.0086	.0030	.0000
J= 15	.2365	.1824	.1352	.0947	.0613	.0358	.0184	.0081	.0027	.0000
J= 14	.2652	.2016	.1461	.0996	.0626	.0356	.0178	.0077	.0026	.0000
J= 13	.2767	.2099	.1508	.1014	.0628	.0350	.0173	.0073	.0024	.0000
J= 12	.2813	.2132	.1526	.1018	.0624	.0345	.0168	.0071	.0023	.0000
J= 11	.2831	.2145	.1531	.1017	.0620	.0340	.0165	.0069	.0022	.0000
J= 10	.2838	.2149	.1532	.1015	.0616	.0336	.0162	.0067	.0022	.0000
J= 9	.2839	.2149	.1530	.1012	.0612	.0333	.0160	.0066	.0022	.0000
J= 8	.2838	.2147	.1528	.1009	.0609	.0331	.0158	.0066	.0021	.0000
J= 7	.2837	.2146	.1526	.1007	.0608	.0329	.0158	.0065	.0021	.0000
J= 6	.2837	.2146	.1526	.1006	.0607	.0329	.0157	.0065	.0021	.0000
J= 5	.2837	.2145	.1525	.1006	.0606	.0328	.0157	.0065	.0021	.0000
J= 4	.2836	.2145	.1524	.1005	.0605	.0328	.0156	.0065	.0021	.0000
J= 3	.2836	.2145	.1524	.1005	.0605	.0327	.0156	.0065	.0021	.0000
J= 2	.2836	.2145	.1524	.1005	.0605	.0327	.0156	.0065	.0021	.0000
J= 1	.2836	.2145	.1524	.1005	.0605	.0327	.0156	.0065	.0021	.0000

$k = 800$ $R'_a = 3.87498$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0093	.0093	.0093	.0093	.0093	.0093	.0093	.0093	.0093	.0093	.0093
J= 20	.0216	.0216	.0216	.0216	.0216	.0216	.0217	.0217	.0216	.0216	.0216
J= 19	.0406	.0406	.0406	.0406	.0406	.0407	.0407	.0406	.0406	.0405	.0404
J= 18	.0712	.0712	.0712	.0712	.0712	.0712	.0712	.0711	.0709	.0706	.0700
J= 17	.1241	.1241	.1241	.1240	.1240	.1240	.1238	.1234	.1222	.1211	.1184
J= 16	.2392	.2392	.2392	.2391	.2386	.2374	.2351	.2311	.2211	.2139	.1990
J= 15	1.0000	1.0000	.9310	.8621	.7938	.7050	.6179	.5336	.4276	.3786	.3104
J= 14	1.0000	1.0000	.9390	.8779	.8169	.7368	.6570	.5775	.4726	.4221	.3483
J= 13	1.0000	1.0000	.9409	.8817	.8225	.7448	.6673	.5898	.4869	.4368	.3626
J= 12	1.0000	1.0000	.9415	.8830	.8245	.7477	.6709	.5943	.4923	.4426	.3685
J= 11	1.0000	1.0000	.9417	.8834	.8251	.7485	.6721	.5959	.4945	.4449	.3710
J= 10	1.0000	1.0000	.9416	.8832	.8248	.7483	.6720	.5961	.4952	.4459	.3721
J= 9	1.0000	1.0000	.9411	.8822	.8235	.7470	.6712	.5959	.4955	.4463	.3726
J= 8	1.0000	1.0000	.9391	.8791	.8204	.7449	.6702	.5955	.4955	.4464	.3728
J= 7	1.0000	1.0000	.9324	.8731	.8168	.7435	.6696	.5953	.4955	.4465	.3729
J= 6	.9258	.9258	.9038	.8633	.8136	.7424	.6692	.5952	.4955	.4465	.3729
J= 5	1.0000	.9812	.9215	.8671	.8136	.7419	.6689	.5951	.4955	.4465	.3729
J= 4	1.0000	.9841	.9259	.8694	.8141	.7418	.6688	.5951	.4955	.4465	.3729
J= 3	1.0000	.9845	.9272	.8705	.8145	.7417	.6687	.5950	.4955	.4465	.3730
J= 2	1.0000	.9845	.9275	.8708	.8146	.7417	.6687	.5950	.4955	.4465	.3730
J= 1	1.0000	.9845	.9272	.8705	.8145	.7417	.6687	.5950	.4955	.4465	.3730

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0093	.0092	.0091	.0089	.0085	.0078	.0066	.0047	.0024	.0000
J= 20	.0215	.0213	.0209	.0202	.0189	.0167	.0132	.0088	.0042	.0000
J= 19	.0401	.0394	.0382	.0360	.0322	.0266	.0194	.0118	.0052	.0000
J= 18	.0688	.0666	.0627	.0563	.0472	.0359	.0240	.0135	.0056	.0000
J= 17	.1135	.1057	.0940	.0787	.0608	.0427	.0267	.0142	.0057	.0000
J= 16	.1790	.1543	.1264	.0976	.0704	.0467	.0279	.0144	.0056	.0000
J= 15	.2498	.1965	.1500	.1097	.0758	.0486	.0283	.0143	.0054	.0000
J= 14	.2794	.2169	.1622	.1160	.0784	.0494	.0283	.0141	.0053	.0000
J= 13	.2917	.2262	.1680	.1189	.0795	.0496	.0282	.0139	.0052	.0000
J= 12	.2971	.2304	.1707	.1203	.0800	.0496	.0280	.0138	.0051	.0000
J= 11	.2995	.2323	.1719	.1208	.0801	.0495	.0279	.0136	.0051	.0000
J= 10	.3006	.2331	.1724	.1210	.0801	.0494	.0278	.0136	.0050	.0000
J= 9	.3010	.2335	.1726	.1211	.0801	.0493	.0277	.0135	.0050	.0000
J= 8	.3012	.2336	.1727	.1210	.0800	.0493	.0276	.0134	.0050	.0000
J= 7	.3013	.2336	.1727	.1210	.0800	.0492	.0276	.0134	.0050	.0000
J= 6	.3013	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0050	.0000
J= 5	.3013	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0050	.0000
J= 4	.3013	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0049	.0000
J= 3	.3013	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0049	.0000
J= 2	.3014	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0049	.0000
J= 1	.3013	.2337	.1727	.1210	.0799	.0492	.0275	.0134	.0049	.0000

$k = 1600$ $R'_a = 3.97388$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0103	.0103	.0103	.0103	.0103	.0103	.0103	.0103	.0103	.0103	.0102
J= 20	.0236	.0236	.0236	.0236	.0236	.0236	.0236	.0236	.0236	.0236	.0235
J= 19	.0436	.0436	.0436	.0436	.0436	.0436	.0436	.0436	.0435	.0434	.0433
J= 18	.0749	.0749	.0749	.0749	.0749	.0749	.0749	.0748	.0746	.0744	.0737
J= 17	.1283	.1283	.1283	.1283	.1283	.1282	.1280	.1277	.1265	.1254	.1226
J= 16	.2435	.2435	.2435	.2433	.2429	.2417	.2394	.2353	.2254	.2183	.2034
J= 15	1.0000	1.0000	.9314	.8630	.7950	.7068	.6203	.5364	.4310	.3824	.3146
J= 14	1.0000	1.0000	.9394	.8787	.8181	.7386	.6592	.5803	.4761	.4259	.3526
J= 13	1.0000	1.0000	.9413	.8825	.8237	.7466	.6696	.5927	.4905	.4407	.3670
J= 12	1.0000	1.0000	.9419	.8838	.8258	.7495	.6733	.5973	.4960	.4466	.3731
J= 11	1.0000	1.0000	.9422	.8842	.8264	.7504	.6746	.5989	.4982	.4491	.3757
J= 10	1.0000	1.0000	.9421	.8841	.8261	.7502	.6745	.5992	.4991	.4501	.3769
J= 9	1.0000	1.0000	.9415	.8831	.8249	.7490	.6737	.5990	.4993	.4505	.3774
J= 8	1.0000	1.0000	.9396	.8800	.8218	.7469	.6727	.5986	.4994	.4507	.3777
J= 7	1.0000	1.0000	.9330	.8741	.8182	.7455	.6722	.5985	.4994	.4508	.3778
J= 6	.9264	.9264	.9046	.8644	.8151	.7445	.6718	.5984	.4994	.4508	.3778
J= 5	1.0000	.9814	.9221	.8682	.8150	.7440	.6715	.5983	.4994	.4508	.3778
J= 4	1.0000	.9842	.9265	.8704	.8155	.7438	.6714	.5982	.4994	.4509	.3779
J= 3	1.0000	.9846	.9278	.8715	.8159	.7437	.6713	.5982	.4994	.4509	.3779
J= 2	1.0000	.9847	.9281	.8718	.8161	.7437	.6713	.5982	.4994	.4509	.3779
J= 1	1.0000	.9846	.9278	.8715	.8159	.7437	.6713	.5982	.4994	.4509	.3779

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0102	.0102	.0101	.0098	.0094	.0087	.0074	.0054	.0028	.0000
J= 20	.0234	.0232	.0228	.0221	.0207	.0184	.0148	.0101	.0049	.0000
J= 19	.0429	.0423	.0410	.0388	.0349	.0292	.0218	.0137	.0062	.0000
J= 18	.0725	.0703	.0663	.0600	.0508	.0393	.0271	.0159	.0069	.0000
J= 17	.1178	.1100	.0984	.0830	.0651	.0469	.0304	.0170	.0072	.0000
J= 16	.1835	.1589	.1312	.1025	.0753	.0514	.0321	.0175	.0072	.0000
J= 15	.2543	.2014	.1552	.1151	.0812	.0538	.0329	.0177	.0072	.0000
J= 14	.2841	.2221	.1677	.1217	.0842	.0550	.0332	.0177	.0072	.0000
J= 13	.2967	.2316	.1738	.1249	.0856	.0554	.0333	.0176	.0071	.0000
J= 12	.3022	.2359	.17[illegible]7	.1265	.0863	.0556	.0333	.0176	.0071	.0000
J= 11	.3047	.2380	.1780	.1272	.0866	.0557	.0332	.0175	.0070	.0000
J= 10	.3059	.2389	.1786	.1275	.0867	.0557	.0332	.0175	.0070	.0000
J= 9	.3064	.2393	.1789	.1276	.0867	.0557	.0332	.0175	.0070	.0000
J= 8	.3066	.2395	.1790	.1277	.0867	.0557	.0331	.0174	.0070	.0000
J= 7	.3067	.2396	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 6	.3068	.2396	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 5	.3068	.2397	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 4	.3068	.2397	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 3	.3068	.2397	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 2	.3068	.2397	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000
J= 1	.3068	.2397	.1791	.1277	.0867	.0556	.0331	.0174	.0070	.0000

$k = 2400$ $\qquad$ $R'_a = 4.00528$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0107	.0107	.0107	.0107	.0107	.0107	.0107	.0107	.0107	.0107	.0107
J= 20	.0245	.0245	.0245	.0245	.0245	.0245	.0245	.0245	.0245	.0245	.0244
J= 19	.0449	.0449	.0449	.0449	.0449	.0449	.0449	.0449	.0448	.0448	.0446
J= 18	.0766	.0766	.0766	.0766	.0766	.0766	.0766	.0765	.0762	.0760	.0754
J= 17	.1302	.1302	.1302	.1302	.1302	.1301	.1299	.1295	.1283	.1272	.1245
J= 16	.2453	.2453	.2453	.2451	.2447	.2435	.2412	.2372	.2273	.2202	.2054
J= 15	1.0000	1.0000	.9316	.8633	.7955	.7075	.6213	.5377	.4325	.3840	.3164
J= 14	1.0000	1.0000	.9396	.8791	.8186	.7393	.6602	.5814	.4775	.4275	.3544
J= 13	1.0000	1.0000	.9415	.8828	.8243	.7474	.6706	.5939	.4920	.4424	.3689
J= 12	1.0000	1.0000	.9421	.8842	.8263	.7503	.6743	.5985	.4976	.4483	.3750
J= 11	1.0000	1.0000	.9423	.8846	.8269	.7512	.6756	.6002	.4998	.4508	.3777
J= 10	1.0000	1.0000	.9422	.8844	.8267	.7510	.6756	.6005	.5006	.4518	.3789
J= 9	1.0000	1.0000	.9417	.8835	.8255	.7498	.6748	.6003	.5009	.4523	.3794
J= 8	1.0000	1.0000	.9398	.8804	.8223	.7477	.6738	.5999	.5010	.4525	.3797
J= 7	1.0000	1.0000	.9332	.8745	.8188	.7463	.6732	.5998	.5010	.4526	.3798
J= 6	.9267	.9267	.9049	.8648	.8157	.7453	.6729	.5997	.5011	.4526	.3798
J= 5	1.0000	.9814	.9224	.8686	.8156	.7448	.6726	.5996	.5011	.4526	.3799
J= 4	1.0000	.9842	.9267	.8708	.8161	.7446	.6725	.5995	.5011	.4526	.3799
J= 3	1.0000	.9846	.9280	.8719	.8165	.7446	.6724	.5995	.5011	.4527	.3799
J= 2	1.0000	.9847	.9283	.8722	.8167	.7446	.6724	.5995	.5011	.4527	.3799
J= 1	1.0000	.9846	.9280	.8719	.8165	.7446	.6724	.5995	.5011	.4527	.3799

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0107	.0106	.0105	.0103	.0099	.0091	.0077	.0057	.0030	.0000
J= 20	.0243	.0241	.0237	.0230	.0216	.0192	.0155	.0107	.0052	.0000
J= 19	.0442	.0436	.0423	.0400	.0362	.0304	.0229	.0146	.0067	.0000
J= 18	.0742	.0719	.0680	.0616	.0524	.0409	.0285	.0171	.0076	.0000
J= 17	.1197	.1119	.1003	.0849	.0670	.0487	.0321	.0184	.0080	.0000
J= 16	.1855	.1610	.1333	.1047	.0775	.0536	.0340	.0191	.0081	.0000
J= 15	.2563	.2035	.1574	.1174	.0835	.0561	.0350	.0194	.0082	.0000
J= 14	.2861	.2243	.1701	.1241	.0867	.0574	.0354	.0195	.0082	.0000
J= 13	.2987	.2338	.1762	.1275	.0882	.0580	.0356	.0195	.0082	.0000
J= 12	.3043	.2383	.1792	.1291	.0890	.0583	.0357	.0195	.0082	.0000
J= 11	.3069	.2403	.1806	.1299	.0893	.0584	.0357	.0195	.0082	.0000
J= 10	.3081	.2413	.1812	.1302	.0895	.0584	.0357	.0195	.0081	.0000
J= 9	.3086	.2418	.1815	.1304	.0895	.0585	.0357	.0195	.0081	.0000
J= 8	.3089	.2420	.1817	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 7	.3089	.2421	.1817	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 6	.3090	.2421	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 5	.3090	.2421	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 4	.3091	.2422	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 3	.3091	.2422	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 2	.3091	.2422	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000
J= 1	.3091	.2422	.1818	.1305	.0896	.0585	.0357	.0194	.0081	.0000

$k = 3200$ $R'_a = 4.01837$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0109	.0109	.0109	.0109	.0109	.0109	.0109	.0109	.0109	.0109	.0109
J= 20	.0249	.0249	.0249	.0249	.0249	.0249	.0249	.0249	.0249	.0249	.0249
J= 19	.0455	.0455	.0455	.0455	.0455	.0455	.0455	.0455	.0454	.0454	.0452
J= 18	.0774	.0774	.0774	.0774	.0774	.0774	.0773	.0773	.0770	.0768	.0762
J= 17	.1311	.1311	.1311	.1311	.1311	.1310	.1308	.1304	.1292	.1281	.1254
J= 16	.2461	.2461	.2461	.2460	.2455	.2444	.2421	.2380	.2281	.2210	.2062
J= 15	1.0000	1.0000	.9316	.8635	.7958	.7079	.6217	.5382	.4332	.3847	.3173
J= 14	1.0000	1.0000	.9396	.8792	.8189	.7396	.6606	.5820	.4782	.4282	.3553
J= 13	1.0000	1.0000	.9415	.8830	.8245	.7477	.6710	.5945	.4927	.4431	.3698
J= 12	1.0000	1.0000	.9422	.8843	.8265	.7506	.6748	.5991	.4983	.4491	.3759
J= 11	1.0000	1.0000	.9424	.8848	.8272	.7515	.6760	.6007	.5005	.4516	.3785
J= 10	1.0000	1.0000	.9423	.8846	.8269	.7514	.6760	.6011	.5014	.4526	.3798
J= 9	1.0000	1.0000	.9418	.8836	.8257	.7502	.6753	.6008	.5017	.4531	.3803
J= 8	1.0000	1.0000	.9398	.8806	.8226	.7481	.6743	.6005	.5018	.4533	.3806
J= 7	1.0000	1.0000	.9333	.8747	.8191	.7467	.6737	.6004	.5018	.4534	.3807
J= 6	.9268	.9268	.9050	.8650	.8160	.7457	.6734	.6003	.5018	.4534	.3807
J= 5	1.0000	.9814	.9225	.8688	.8159	.7452	.6731	.6002	.5018	.4534	.3808
J= 4	1.0000	.9843	.9268	.8710	.8164	.7450	.6729	.6001	.5018	.4535	.3808
J= 3	1.0000	.9847	.9281	.8721	.8168	.7450	.6729	.6001	.5018	.4535	.3808
J= 2	1.0000	.9847	.9284	.8724	.8170	.7449	.6728	.6001	.5018	.4535	.3808
J= 1	1.0000	.9847	.9281	.8721	.8168	.7450	.6729	.6001	.5018	.4535	.3808

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0109	.0108	.0107	.0105	.0101	.0093	.0079	.0058	.0031	.0000
J= 20	.0247	.0245	.0242	.0234	.0220	.0196	.0159	.0110	.0054	.0000
J= 19	.0449	.0442	.0429	.0407	.0368	.0310	.0234	.0150	.0070	.0000
J= 18	.0750	.0727	.0688	.0624	.0532	.0416	.0292	.0176	.0079	.0000
J= 17	.1206	.1127	.1012	.0858	.0679	.0496	.0329	.0191	.0084	.0000
J= 16	.1864	.1619	.1343	.1057	.0785	.0546	.0350	.0199	.0086	.0000
J= 15	.2572	.2045	.1584	.1184	.0846	.0572	.0360	.0202	.0087	.0000
J= 14	.2871	.2253	.1711	.1253	.0879	.0586	.0365	.0204	.0087	.0000
J= 13	.2997	.2349	.1773	.1287	.0895	.0592	.0367	.0204	.0087	.0000
J= 12	.3053	.2393	.1803	.1303	.0902	.0595	.0368	.0204	.0087	.0000
J= 11	.3079	.2414	.1817	.1311	.0906	.0597	.0369	.0205	.0087	.0000
J= 10	.3091	.2424	.1824	.1315	.0907	.0597	.0369	.0205	.0087	.0000
J= 9	.3096	.2429	.1827	.1317	.0908	.0598	.0369	.0204	.0087	.0000
J= 8	.3099	.2431	.1829	.1317	.0909	.0598	.0369	.0204	.0087	.0000
J= 7	.3100	.2432	.1829	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 6	.3100	.2432	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 5	.3101	.2433	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 4	.3101	.2433	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 3	.3101	.2433	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 2	.3101	.2433	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000
J= 1	.3101	.2433	.1830	.1318	.0909	.0598	.0369	.0204	.0087	.0000

$k = 4000$ $R'_a = 4.02435$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0110	.0110	.0110
J= 20	.0252	.0252	.0252	.0252	.0252	.0252	.0252	.0251	.0251	.0251	.0251
J= 19	.0458	.0458	.0458	.0458	.0458	.0458	.0458	.0458	.0457	.0457	.0455
J= 18	.0778	.0778	.0778	.0778	.0778	.0778	.0777	.0776	.0774	.0772	.0765
J= 17	.1315	.1315	.1315	.1315	.1315	.1314	.1312	.1308	.1296	.1285	.1258
J= 16	.2465	.2465	.2465	.2464	.2459	.2448	.2425	.2384	.2285	.2215	.2067
J= 15	1.0000	1.0000	.9317	.8636	.7959	.7080	.6219	.5385	.4335	.3851	.3176
J= 14	1.0000	1.0000	.9397	.8793	.8190	.7398	.6608	.5822	.4785	.4286	.3557
J= 13	1.0000	1.0000	.9416	.8831	.8246	.7479	.6712	.5947	.4930	.4435	.3702
J= 12	1.0000	1.0000	.9422	.8844	.8266	.7508	.6750	.5993	.4986	.4494	.3763
J= 11	1.0000	1.0000	.9425	.8848	.8273	.7517	.6763	.6010	.5008	.4519	.3790
J= 10	1.0000	1.0000	.9424	.8847	.8271	.7515	.6763	.6013	.5017	.4530	.3802
J= 9	1.0000	1.0000	.9419	.8837	.8258	.7503	.6755	.6011	.5020	.4535	.3808
J= 8	1.0000	1.0000	.9399	.8807	.8227	.7482	.6745	.6008	.5021	.4537	.3810
J= 7	1.0000	1.0000	.9333	.8748	.8192	.7468	.6740	.6007	.5021	.4537	.3811
J= 6	.9268	.9268	.9051	.8651	.8161	.7458	.6736	.6006	.5022	.4538	.3812
J= 5	1.0000	.9815	.9225	.8689	.8160	.7453	.6733	.6005	.5022	.4538	.3812
J= 4	1.0000	.9843	.9269	.8711	.8165	.7452	.6732	.6004	.5022	.4538	.3813
J= 3	1.0000	.9847	.9282	.8722	.8169	.7451	.6731	.6004	.5022	.4539	.3813
J= 2	1.0000	.9847	.9285	.8725	.8171	.7451	.6731	.6004	.5022	.4539	.3813
J= 1	1.0000	.9847	.9282	.8722	.8169	.7451	.6731	.6004	.5022	.4539	.3813

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0110	.0109	.0108	.0106	.0102	.0094	.0080	.0059	.0031	.0000
J= 20	.0250	.0248	.0244	.0236	.0222	.0198	.0161	.0111	.0055	.0000
J= 19	.0452	.0445	.0432	.0409	.0371	.0313	.0236	.0152	.0071	.0000
J= 18	.0753	.0731	.0691	.0627	.0535	.0420	.0295	.0179	.0081	.0000
J= 17	.1210	.1132	.1016	.0863	.0684	.0500	.0333	.0194	.0086	.0000
J= 16	.1868	.1623	.1347	.1061	.0790	.0550	.0354	.0202	.0088	.0000
J= 15	.2576	.2049	.1589	.1190	.0852	.0577	.0365	.0206	.0089	.0000
J= 14	.2875	.2257	.1716	.1258	.0884	.0591	.0370	.0208	.0090	.0000
J= 13	.3001	.2354	.1779	.1292	.0900	.0598	.0373	.0209	.0090	.0000
J= 12	.3058	.2398	.1809	.1309	.0908	.0601	.0374	.0209	.0090	.0000
J= 11	.3083	.2419	.1823	.1317	.0912	.0603	.0374	.0209	.0090	.0000
J= 10	.3095	.2429	.1830	.1321	.0914	.0603	.0375	.0209	.0090	.0000
J= 9	.3101	.2434	.1833	.1323	.0914	.0604	.0375	.0209	.0090	.0000
J= 8	.3104	.2436	.1835	.1323	.0915	.0604	.0375	.0209	.0090	.0000
J= 7	.3105	.2437	.1835	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 6	.3105	.2438	.1835	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 5	.3106	.2438	.1836	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 4	.3106	.2438	.1836	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 3	.3106	.2438	.1836	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 2	.3106	.2438	.1836	.1324	.0915	.0604	.0375	.0209	.0090	.0000
J= 1	.3106	.2438	.1836	.1324	.0915	.0604	.0375	.0209	.0090	.0000

$k = 4800$ $\qquad$ $R'_a = 4.02719$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252	.0252	.0252	.0252
J= 19	.0460	.0460	.0460	.0460	.0460	.0460	.0460	.0459	.0459	.0458	.0456
J= 18	.0780	.0780	.0780	.0780	.0780	.0779	.0779	.0778	.0776	.0773	.0767
J= 17	.1317	.1317	.1317	.1317	.1317	.1316	.1314	.1310	.1298	.1287	.1260
J= 16	.2467	.2467	.2467	.2466	.2461	.2449	.2427	.2386	.2287	.2217	.2069
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7081	.6220	.5386	.4337	.3853	.3178
J= 14	1.0000	1.0000	.9397	.8793	.8190	.7399	.6609	.5824	.4787	.4288	.3559
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7479	.6713	.5948	.4931	.4437	.3704
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6751	.5995	.4987	.4496	.3765
J= 11	1.0000	1.0000	.9425	.8849	.8273	.7518	.6764	.6011	.5010	.4521	.3792
J= 10	1.0000	1.0000	.9424	.8847	.8271	.7516	.6764	.6015	.5019	.4532	.3804
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7504	.6756	.6012	.5022	.4537	.3810
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7483	.6746	.6009	.5023	.4539	.3812
J= 7	1.0000	1.0000	.9334	.8748	.8193	.7469	.6741	.6008	.5023	.4539	.3813
J= 6	.9269	.9269	.9051	.8652	.8161	.7459	.6737	.6007	.5023	.4540	.3814
J= 5	1.0000	.9815	.9226	.8689	.8161	.7454	.6734	.6006	.5023	.4540	.3814
J= 4	1.0000	.9843	.9269	.8712	.8166	.7453	.6733	.6006	.5023	.4540	.3815
J= 3	1.0000	.9847	.9282	.8722	.8170	.7452	.6732	.6005	.5023	.4540	.3815
J= 2	1.0000	.9847	.9285	.8725	.8172	.7452	.6732	.6005	.5023	.4540	.3815
J= 1	1.0000	.9847	.9282	.8722	.8170	.7452	.6732	.6005	.5023	.4540	.3815

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0110	.0110	.0109	.0106	.0102	.0094	.0080	.0059	.0031	.0000
J= 20	.0251	.0248	.0245	.0237	.0223	.0199	.0161	.0112	.0056	.0000
J= 19	.0453	.0446	.0434	.0411	.0372	.0314	.0238	.0153	.0072	.0000
J= 18	.0755	.0733	.0693	.0629	.0537	.0421	.0297	.0180	.0082	.0000
J= 17	.1212	.1134	.1018	.0865	.0686	.0502	.0335	.0196	.0087	.0000
J= 16	.1870	.1626	.1349	.1064	.0792	.0553	.0356	.0204	.0089	.0000
J= 15	.2578	.2052	.1591	.1192	.0854	.0580	.0367	.0208	.0091	.0000
J= 14	.2877	.2260	.1719	.1261	.0887	.0594	.0373	.0210	.0091	.0000
J= 13	.3004	.2356	.1781	.1295	.0903	.0601	.0375	.0211	.0091	.0000
J= 12	.3060	.2401	.1811	.1312	.0911	.0604	.0377	.0211	.0092	.0000
J= 11	.3085	.2422	.1826	.1320	.0915	.0606	.0377	.0212	.0092	.0000
J= 10	.3098	.2432	.1832	.1324	.0917	.0606	.0377	.0212	.0092	.0000
J= 9	.3103	.2437	.1836	.1325	.0917	.0607	.0378	.0212	.0092	.0000
J= 8	.3106	.2439	.1837	.1326	.0918	.0607	.0378	.0212	.0092	.0000
J= 7	.3107	.2440	.1838	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 6	.3107	.2440	.1838	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 5	.3108	.2440	.1838	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 4	.3108	.2441	.1839	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 3	.3108	.2441	.1839	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 2	.3108	.2441	.1839	.1327	.0918	.0607	.0378	.0212	.0092	.0000
J= 1	.3108	.2441	.1839	.1327	.0918	.0607	.0378	.0212	.0092	.0000

$k = 5600$ $R'_a = 4.02855$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252
J= 19	.0461	.0461	.0461	.0461	.0461	.0461	.0460	.0460	.0460	.0459	.0457
J= 18	.0781	.0781	.0781	.0780	.0780	.0780	.0780	.0779	.0777	.0774	.0768
J= 17	.1318	.1318	.1318	.1318	.1318	.1317	.1315	.1311	.1299	.1288	.1261
J= 16	.2468	.2468	.2468	.2467	.2462	.2450	.2428	.2387	.2288	.2217	.2070
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7082	.6221	.5387	.4338	.3853	.3179
J= 14	1.0000	1.0000	.9397	.8794	.8190	.7399	.6610	.5824	.4788	.4288	.3560
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7480	.6714	.5949	.4932	.4438	.3705
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6751	.5995	.4988	.4497	.3766
J= 11	1.0000	1.0000	.9425	.8849	.8274	.7518	.6764	.6012	.5011	.4522	.3793
J= 10	1.0000	1.0000	.9424	.8847	.8271	.7516	.6764	.6015	.5019	.4533	.3805
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7505	.6756	.6013	.5023	.4537	.3811
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7484	.6746	.6010	.5024	.4539	.3813
J= 7	1.0000	1.0000	.9334	.8748	.8193	.7470	.6741	.6009	.5024	.4540	.3814
J= 6	.9269	.9269	.9052	.8652	.8162	.7460	.6738	.6008	.5024	.4541	.3815
J= 5	1.0000	.9815	.9226	.8689	.8161	.7455	.6735	.6007	.5024	.4541	.3815
J= 4	1.0000	.9843	.9269	.8712	.8166	.7453	.6733	.6006	.5024	.4541	.3816
J= 3	1.0000	.9847	.9282	.8722	.8170	.7453	.6733	.6006	.5024	.4541	.3816
J= 2	1.0000	.9847	.9285	.8725	.8172	.7453	.6732	.6006	.5024	.4541	.3816
J= 1	1.0000	.9847	.9282	.8722	.8170	.7453	.6733	.6006	.5024	.4541	.3816

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0110	.0109	.0107	.0102	.0094	.0081	.0059	.0031	.0000
J= 20	.0251	.0249	.0245	.0237	.0223	.0199	.0162	.0112	.0056	.0000
J= 19	.0454	.0447	.0434	.0412	.0373	.0315	.0238	.0154	.0072	.0000
J= 18	.0756	.0733	.0694	.0630	.0538	.0422	.0298	.0181	.0082	.0000
J= 17	.1213	.1135	.1019	.0866	.0687	.0503	.0336	.0197	.0087	.0000
J= 16	.1871	.1627	.1350	.1065	.0793	.0554	.0357	.0205	.0090	.0000
J= 15	.2579	.2053	.1592	.1193	.0855	.0581	.0368	.0209	.0091	.0000
J= 14	.2878	.2261	.1720	.1262	.0888	.0595	.0374	.0211	.0092	.0000
J= 13	.3005	.2357	.1782	.1296	.0904	.0602	.0377	.0212	.0092	.0000
J= 12	.3061	.2402	.18[illegible]3	.1313	.0912	.0605	.0378	.0212	.0092	.0000
J= 11	.3087	.2423	.1827	.1321	.0916	.0607	.0378	.0213	.0092	.0000
J= 10	.3099	.2433	.1834	.1325	.0918	.0608	.0379	.0213	.0092	.0000
J= 9	.3104	.2438	.1837	.1327	.0919	.0608	.0379	.0213	.0092	.0000
J= 8	.3107	.2440	.1839	.1328	.0919	.0608	.0379	.0213	.0092	.0000
J= 7	.3108	.2441	.1839	.1328	.0919	.0608	.0379	.0213	.0092	.0000
J= 6	.3109	.2441	.1839	.1328	.0919	.0608	.0379	.0213	.0092	.0000
J= 5	.3109	.2442	.1840	.1328	.0920	.0608	.0379	.0213	.0092	.0000
J= 4	.3109	.2442	.1840	.1328	.0920	.0608	.0379	.0213	.0092	.0000
J= 3	.3109	.2442	.1840	.1328	.0920	.0608	.0379	.0213	.0092	.0000
J= 2	.3110	.2442	.1840	.1329	.0920	.0608	.0379	.0213	.0092	.0000
J= 1	.3109	.2442	.1840	.1328	.0920	.0608	.0379	.0213	.0092	.0000

$k = 6400$ $R'_a = 4.02920$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252
J= 19	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0460	.0459	.0458
J= 18	.0781	.0781	.0781	.0781	.0781	.0781	.0780	.0780	.0777	.0775	.0768
J= 17	.1319	.1319	.1319	.1319	.1318	.1317	.1315	.1312	.1299	.1289	.1261
J= 16	.2469	.2469	.2469	.2467	.2463	.2451	.2428	.2388	.2289	.2218	.2070
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7082	.6221	.5387	.4338	.3854	.3180
J= 14	1.0000	1.0000	.9397	.8794	.8191	.7399	.6610	.5824	.4788	.4289	.3560
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7480	.6714	.5949	.4932	.4438	.3705
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6752	.5996	.4989	.4497	.3766
J= 11	1.0000	1.0000	.9425	.8849	.8274	.7519	.6764	.6012	.5011	.4523	.3793
J= 10	1.0000	1.0000	.9424	.8847	.8272	.7517	.6764	.6016	.5020	.4533	.3805
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7505	.6757	.6013	.5023	.4538	.3811
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7484	.6747	.6010	.5024	.4540	.3814
J= 7	1.0000	1.0000	.9334	.8749	.8193	.7470	.6741	.6009	.5024	.4541	.3815
J= 6	.9269	.9269	.9052	.8652	.8162	.7460	.6738	.6008	.5024	.4541	.3815
J= 5	1.0000	.9815	.9226	.8690	.8161	.7455	.6735	.6007	.5025	.4541	.3816
J= 4	1.0000	.9843	.9269	.8712	.8167	.7453	.6734	.6007	.5025	.4542	.3816
J= 3	1.0000	.9847	.9282	.8722	.8171	.7453	.6733	.6006	.5025	.4542	.3816
J= 2	1.0000	.9848	.9285	.8725	.8172	.7453	.6733	.6006	.5025	.4542	.3816
J= 1	1.0000	.9847	.9282	.8722	.8171	.7453	.6733	.6006	.5025	.4542	.3816

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0110	.0109	.0107	.0102	.0094	.0081	.0059	.0031	.0000
J= 20	.0251	.0249	.0245	.0238	.0223	.0199	.0162	.0112	.0056	.0000
J= 19	.0454	.0447	.0435	.0412	.0373	.0315	.0239	.0154	.0073	.0000
J= 18	.0756	.0734	.0694	.0630	.0538	.0423	.0298	.0181	.0082	.0000
J= 17	.1213	.1135	.1019	.0866	.0687	.0504	.0336	.0197	.0088	.0000
J= 16	.1872	.1627	.1351	.1065	.0794	.0554	.0358	.0205	.0090	.0000
J= 15	.2580	.2053	.1593	.1194	.0856	.0582	.0369	.0210	.0091	.0000
J= 14	.2879	.2261	.1721	.1263	.0889	.0596	.0375	.0212	.0092	.0000
J= 13	.3005	.2358	.1783	.1297	.0905	.0603	.0377	.0213	.0092	.0000
J= 12	.3061	.2402	.1813	.1314	.0913	.0606	.0379	.0213	.0093	.0000
J= 11	.3087	.2424	.1828	.1322	.0917	.0608	.0379	.0213	.0093	.0000
J= 10	.3099	.2434	.1834	.1326	.0919	.0608	.0379	.0213	.0093	.0000
J= 9	.3105	.2438	.1838	.1328	.0920	.0609	.0380	.0213	.0093	.0000
J= 8	.3108	.2441	.1839	.1328	.0920	.0609	.0380	.0213	.0093	.0000
J= 7	.3109	.2441	.1840	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 6	.3109	.2442	.1840	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 5	.3110	.2442	.1840	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 4	.3110	.2443	.1841	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 3	.3110	.2443	.1841	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 2	.3110	.2443	.1841	.1329	.0920	.0609	.0380	.0213	.0093	.0000
J= 1	.3110	.2443	.1841	.1329	.0920	.0609	.0380	.0213	.0093	.0000

$k = 7200$ $\qquad$ $R'_a = 4.02952$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252
J= 19	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0460	.0459	.0458
J= 18	.0781	.0781	.0781	.0781	.0781	.0781	.0781	.0780	.0777	.0775	.0769
J= 17	.1319	.1319	.1319	.1319	.1318	.1318	.1316	.1312	.1300	.1289	.1261
J= 16	.2469	.2469	.2469	.2467	.2463	.2451	.2428	.2388	.2289	.2218	.2070
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7082	.6221	.5387	.4338	.3854	.3180
J= 14	1.0000	1.0000	.9397	.8794	.8191	.7399	.6610	.5825	.4788	.4289	.3560
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7480	.6714	.5949	.4933	.4438	.3705
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6752	.5996	.4989	.4498	.3766
J= 11	1.0000	1.0000	.9425	.8849	.8274	.7519	.6765	.6012	.5011	.4523	.3793
J= 10	1.0000	1.0000	.9424	.8848	.8272	.7517	.6764	.6016	.5020	.4533	.3806
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7505	.6757	.6014	.5023	.4538	.3811
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7484	.6747	.6011	.5024	.4540	.3814
J= 7	1.0000	1.0000	.9334	.8749	.8193	.7470	.6742	.6009	.5024	.4541	.3815
J= 6	.9269	.9269	.9052	.8652	.8162	.7460	.6738	.6008	.5025	.4541	.3816
J= 5	1.0000	.9815	.9226	.8690	.8162	.7455	.6735	.6007	.5025	.4542	.3816
J= 4	1.0000	.9843	.9269	.8712	.8167	.7453	.6734	.6007	.5025	.4542	.3816
J= 3	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6006	.5025	.4542	.3816
J= 2	1.0000	.9848	.9285	.8726	.8172	.7453	.6733	.6006	.5025	.4542	.3817
J= 1	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6006	.5025	.4542	.3816

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0110	.0109	.0107	.0103	.0095	.0081	.0059	.0031	.0000
J= 20	.0251	.0249	.0245	.0238	.0224	.0199	.0162	.0112	.0056	.0000
J= 19	.0454	.0448	.0435	.0412	.0373	.0315	.0239	.0154	.0073	.0000
J= 18	.0757	.0734	.0695	.0631	.0538	.0423	.0298	.0182	.0082	.0000
J= 17	.1213	.1135	.1020	.0866	.0687	.0504	.0337	.0197	.0088	.0000
J= 16	.1872	.1627	.1351	.1066	.0794	.0555	.0358	.0206	.0090	.0000
J= 15	.2580	.2054	.1593	.1194	.0856	.0582	.0369	.0210	.0092	.0000
J= 14	.2879	.2262	.1721	.1263	.0889	.0596	.0375	.0212	.0092	.0000
J= 13	.3005	.2358	.1783	.1297	.0905	.0603	.0378	.0213	.0093	.0000
J= 12	.3062	.2403	.1813	.1314	.0913	.0606	.0379	.0213	.0093	.0000
J= 11	.3087	.2424	.1828	.1322	.0917	.0608	.0379	.0213	.0093	.0000
J= 10	.3099	.2434	.1835	.1326	.0919	.0609	.0380	.0214	.0093	.0000
J= 9	.3105	.2439	.1838	.1328	.0920	.0609	.0380	.0214	.0093	.0000
J= 8	.3108	.2441	.1840	.1329	.0920	.0609	.0380	.0214	.0093	.0000
J= 7	.3109	.2442	.1840	.1329	.0920	.0609	.0380	.0214	.0093	.0000
J= 6	.3109	.2442	.1840	.1329	.0921	.0609	.0380	.0214	.0093	.0000
J= 5	.3110	.2443	.1841	.1329	.0921	.0609	.0380	.0214	.0093	.0000
J= 4	.3110	.2443	.1841	.1329	.0921	.0609	.0380	.0214	.0093	.0000
J= 3	.3110	.2443	.1841	.1330	.0921	.0609	.0380	.0214	.0093	.0000
J= 2	.3110	.2443	.1841	.1330	.0921	.0609	.0380	.0214	.0093	.0000
J= 1	.3110	.2443	.1841	.1330	.0921	.0609	.0380	.0214	.0093	.0000

$k = 8000$ $R'_a = 4.02967$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0112	.0112	.0112	.0112	.0112	.0112	.0112	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252
J= 19	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0460	.0459	.0458
J= 18	.0781	.0781	.0781	.0781	.0781	.0781	.0781	.0780	.0777	.0775	.0769
J= 17	.1319	.1319	.1319	.1319	.1319	.1318	.1316	.1312	.1300	.1289	.1262
J= 16	.2469	.2469	.2469	.2467	.2463	.2451	.2428	.2388	.2289	.2218	.2070
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7082	.6221	.5387	.4338	.3854	.3180
J= 14	1.0000	1.0000	.9397	.8794	.8191	.7399	.6610	.5825	.4788	.4289	.3560
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7480	.6714	.5950	.4933	.4438	.3705
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6752	.5996	.4989	.4498	.3766
J= 11	1.0000	1.0000	.9425	.8849	.8274	.7519	.6765	.6012	.5012	.4523	.3793
J= 10	1.0000	1.0000	.9424	.8848	.8272	.7517	.6765	.6016	.5020	.4534	.3806
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7505	.6757	.6014	.5023	.4538	.3812
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7484	.6747	.6011	.5024	.4540	.3814
J= 7	1.0000	1.0000	.9334	.8749	.8193	.7470	.6742	.6009	.5025	.4541	.3815
J= 6	.9269	.9269	.9052	.8652	.8162	.7460	.6738	.6008	.5025	.4541	.3816
J= 5	1.0000	.9815	.9226	.8690	.8162	.7455	.6735	.6007	.5025	.4542	.3816
J= 4	1.0000	.9843	.9269	.8712	.8167	.7453	.6734	.6007	.5025	.4542	.3816
J= 3	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6006	.5025	.4542	.3817
J= 2	1.0000	.9848	.9285	.8726	.8172	.7453	.6733	.6006	.5025	.4542	.3817
J= 1	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6006	.5025	.4542	.3817

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0110	.0109	.0107	.0103	.0095	.0081	.0059	.0031	.0000
J= 20	.0251	.0249	.0245	.0238	.0224	.0199	.0162	.0112	.0056	.0000
J= 19	.0454	.0448	.0435	.0412	.0373	.0315	.0239	.0154	.0073	.0000
J= 18	.0757	.0734	.0695	.0631	.0539	.0423	.0298	.0182	.0082	.0000
J= 17	.1213	.1135	.1020	.0866	.0688	.0504	.0337	.0197	.0088	.0000
J= 16	.1872	.1627	.1351	.1066	.0794	.0555	.0358	.0206	.0090	.0000
J= 15	.2580	.2054	.1593	.1194	.0856	.0582	.0369	.0210	.0092	.0000
J= 14	.2879	.2262	.1721	.1263	.0889	.0596	.0375	.0212	.0092	.0000
J= 13	.3005	.2358	.1784	.1297	.0905	.0603	.0378	.0213	.0093	.0000
J= 12	.3062	.2403	.1814	.1314	.0913	.0607	.0379	.0213	.0093	.0000
J= 11	.3087	.2424	.1828	.1322	.0917	.0608	.0380	.0214	.0093	.0000
J= 10	.3100	.2434	.1835	.1326	.0919	.0609	.0380	.0214	.0093	.0000
J= 9	.3105	.2439	.1838	.1328	.0920	.0609	.0380	.0214	.0093	.0000
J= 8	.3108	.2441	.1840	.1329	.0920	.0609	.0380	.0214	.0093	.0000
J= 7	.3109	.2442	.1840	.1329	.0921	.0609	.0380	.0214	.0093	.0000
J= 6	.3110	.2442	.1841	.1329	.0921	.0610	.0380	.0214	.0093	.0000
J= 5	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 4	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 3	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 2	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 1	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000

$k = 8800$ $\qquad$ $R'_a = 4.02975$

I=	1	2	3	4	5	6	7	8	9	10	11
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0112	.0112	.0112	.0112	.0112	.0112	.0112	.0111	.0111	.0111	.0111
J= 20	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0253	.0252
J= 19	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0461	.0460	.0460	.0458
J= 18	.0781	.0781	.0781	.0781	.0781	.0781	.0781	.0780	.0777	.0775	.0769
J= 17	.1319	.1319	.1319	.1319	.1319	.1318	.1316	.1312	.1300	.1289	.1262
J= 16	.2469	.2469	.2469	.2467	.2463	.2451	.2428	.2388	.2289	.2218	.2070
J= 15	1.0000	1.0000	.9317	.8636	.7960	.7082	.6221	.5387	.4338	.3854	.3180
J= 14	1.0000	1.0000	.9397	.8794	.8191	.7399	.6610	.5825	.4788	.4289	.3560
J= 13	1.0000	1.0000	.9416	.8831	.8247	.7480	.6714	.5950	.4933	.4438	.3705
J= 12	1.0000	1.0000	.9423	.8845	.8267	.7509	.6752	.5996	.4989	.4498	.3766
J= 11	1.0000	1.0000	.9425	.8849	.8274	.7519	.6765	.6013	.5012	.4523	.3793
J= 10	1.0000	1.0000	.9424	.8848	.8272	.7517	.6765	.6016	.5020	.4534	.3806
J= 9	1.0000	1.0000	.9419	.8838	.8259	.7505	.6757	.6014	.5023	.4538	.3812
J= 8	1.0000	1.0000	.9399	.8807	.8228	.7484	.6747	.6011	.5024	.4540	.3814
J= 7	1.0000	1.0000	.9334	.8749	.8193	.7470	.6742	.6009	.5025	.4541	.3815
J= 6	.9269	.9269	.9052	.8652	.8162	.7460	.6738	.6008	.5025	.4541	.3816
J= 5	1.0000	.9815	.9226	.8690	.8162	.7455	.6735	.6007	.5025	.4542	.3816
J= 4	1.0000	.9843	.9269	.8712	.8167	.7453	.6734	.6007	.5025	.4542	.3816
J= 3	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6007	.5025	.4542	.3817
J= 2	1.0000	.9848	.9285	.8726	.8172	.7453	.6733	.6006	.5025	.4542	.3817
J= 1	1.0000	.9847	.9282	.8723	.8171	.7453	.6733	.6007	.5025	.4542	.3817

I=	12	13	14	15	16	17	18	19	20	21
J= 22	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
J= 21	.0111	.0110	.0109	.0107	.0103	.0095	.0081	.0059	.0031	.0000
J= 20	.0251	.0249	.0245	.0238	.0224	.0200	.0162	.0112	.0056	.0000
J= 19	.0454	.0448	.0435	.0412	.0373	.0315	.0239	.0154	.0073	.0000
J= 18	.0757	.0734	.0695	.0631	.0539	.0423	.0298	.0182	.0082	.0000
J= 17	.1214	.1135	.1020	.0866	.0688	.0504	.0337	.0198	.0088	.0000
J= 16	.1872	.1628	.1351	.1066	.0794	.0555	.0358	.0206	.0090	.0000
J= 15	.2580	.2054	.1593	.1194	.0856	.0582	.0370	.0210	.0092	.0000
J= 14	.2879	.2262	.1721	.1263	.0889	.0596	.0375	.0212	.0092	.0000
J= 13	.3005	.2358	.1784	.1297	.0906	.0603	.0378	.0213	.0093	.0000
J= 12	.3062	.2403	.1814	.1314	.0914	.0607	.0379	.0213	.0093	.0000
J= 11	.3088	.2424	.1828	.1322	.0917	.0608	.0380	.0214	.0093	.0000
J= 10	.3100	.2434	.1835	.1326	.0919	.0609	.0380	.0214	.0093	.0000
J= 9	.3105	.2439	.1838	.1328	.0920	.0609	.0380	.0214	.0093	.0000
J= 8	.3108	.2441	.1840	.1329	.0921	.0610	.0380	.0214	.0093	.0000
J= 7	.3109	.2442	.1840	.1329	.0921	.0610	.0380	.0214	.0093	.0000
J= 6	.3110	.2442	.1841	.1329	.0921	.0610	.0380	.0214	.0093	.0000
J= 5	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 4	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 3	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 2	.3111	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000
J= 1	.3110	.2443	.1841	.1330	.0921	.0610	.0380	.0214	.0093	.0000

$k = 9600$ $R'_a = 4.62978$

VITA

The author was born in Houston, Texas the son of Hughes Mead Zenor and Alpha Erle Zenor. He attended public schools in Houston, Texas and Tulsa, Oklahoma, and enrolled at both Benedictine Heights College and the University of Tulsa in 1958. After one year he transferred to the University of Missouri — Rolla (then the Missouri School of Mines and Metallurgy) and received a B.S. degree in Applied Mathematics in 1963. He fulfilled the last semester's requirements by attending the evening division of the Illinois Institute of Technology while he worked at Argonne National Laboratory in Lamont, Illinois.

After spending one and one-half years in the Department of Mathematical Biology at the University of Chicago, he returned to the University of Missouri — Rolla to complete the requirements for his graduate degrees from that school; he received the M.S. degree in Computer Science in May, 1966 and is due to receive the Ph.D. degree in Geophysical Engineering in May, 1968.

In June 1966 he began work at the Naval Weapons Center, China Lake, California where he has been working in the field of navigation, guidance, and control. During this period he completed the research for his doctorate from the University of Missouri — Rolla.